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A SEPARABLE PROGRAMMING APPROACH TO ALLOCATING MANPOWER FOR THE CALIFORNIA HIGHWAY PATROL

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THESIS

A SEPARABLE PROGRAMMING APPROACH TO
ALLOCATING MANPOWER FOR THE
CALIFORNIA HIGHWAY PATROL

by

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March 1973

A Separable Programming Approach

to

Allocating Manpower for the California Highway Patrol

by

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ABSTRACT

This paper presents a mathematical approach to formulating a problem in the allocation of manpower for the Monterey County Area of the California Highway Patrol (CHP). The technique employed is to formulate the manpower allocation problem of the CHP as a nonlinear programming problem. The problem turns out to be separable, and the Mathematical Programming System/360 for the IBM 360 computer is subsequently used. This project was undertaken with the cooperation of the CHP.

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I. INTRODUCTION

The California Highway Patrol (CHP) has the primary duty of reducing automobile accidents which result in property damage, bodily injury, or death. The manner in which they approach this task is through the employment of patrol officers throughout various areas of responsibility. The patrol officers issue warnings and citations to violators of traffic laws, inspect the mechanical conditions of automobiles, and assist those involved in traffic accidents.

The criterion used for allocation of manpower by the CHP is based on the idea that manpower strength should be tailored to the accident rate over the day by hour of the day [Ref.s 1 and 2].

Reports of all accidents and traffic violations are filed with the CHP Headquarters in Sacramento, California. There the reports are tabulated and published quarterly in the "Accident Enforcement and Patrol Summary" (AEPS), which is forwarded to the local areas. The distribution of accidents are obtained from the AEPS and are used as the basis for determining the required assignment of officers. To meet the required assignment, the Areas distribute the officers among shifts during the day with each officer working an eight-and-one-half-hour shift under normal circumstances.

To determine the optimal number of officers to assign to each shift, the CHP utilizes computations described on page 2 of Appendix A. A close examination of these computations reveals that they are only valid when considering a three-shift day with no overlap of shifts. When more than three shifts are considered, and therefore the shifts overlap, the methodology breaks down due to the inability to account for the effect of interaction between shifts. However, if the procedure in Appendix A is used, it does provide a starting point from which refinements can then be made.

Presently refinements are made through trial and error. To illustrate, an assignment is made and the local area operates for a period of time. When the AEPS is received from Sacramento, a close examination is made to determine the weak areas and more adjustments are made. The feedback cycle for these adjustments is on the order of six or more months.

To assist the CHP in obtaining a method to solve the problem of allocation, it was decided to model the constraints utilizing nonlinear programming techniques. The nonlinear problem evolved into one which was piecewise linear and therefore became suited to separable programming techniques. In separable form it was then formatted to fit the Mathematical Programming System (MPS)/360 for the IBM 360 computer which was used to solve the problem.

In examining data available at the Monterey County Area, a secondary benefit evolved which resulted in a

closer examination of accident rates and distributions. No analytical techniques were involved in this effort, but closer examination of the accident trends in various areas of the county evolved into more accurate descriptions of accident distributions.

II. PROBLEM DEFINITION

This portion of the paper is divided into two sections. The first examines the constraints that were considered in selecting a model for the manpower allocation problem and the second considers the data available for studying trends in accident statistics.

A. CONSTRAINTS CONSIDERED

One of the major problems which face the CHP in solving the problem of manpower allocation is the large number of constraints which must be considered. Through various interviews with the CHP, it was determined that many of the factors considered by scheduling officers while allocating their men are not suitable for an analytical approach. It therefore became necessary to isolate, as much as possible, those factors which needed to be considered utilizing management techniques from the constraints that would be applicable to a mathematical model. The applicable constraints are discussed in the following paragraphs.

The Monterey Area has 56 officers available for assignment. This number takes into account the officers who are on limited vacations. When the officers are on duty, they work 20 to 21 days per month on a five-day work week with two consecutive days off.

The Monterey Area requires that each hour of the day be covered by at least one shift. This requires that there be

a minimum of three shifts per day. For administrative reasons, the maximum number of shifts allowed for one day is five.

In order to meet the scheduling criterion stated in the introduction, the scheduling of officers must consider two factors: the occurrence of accidents varies with the day of the week (Figure 1), and the distribution of accidents over a day varies by hour of the day (Figure 2). The officers are thereby scheduled accordingly. The scheduling is accomplished on a monthly basis and, barring unforeseen circumstances, remains fixed for that period of time.

The number of patrol cars available is dependent upon the day of the week and the hour of day. On normal working days, Monday through Friday, there are approximately 18 patrol cars available between 0800 and 1700 hours. On weekends, holidays, and after 1700 hours on normal working days there are approximately 20 patrol cars available.

Other considerations taken into account are that an officer working alone after dark makes fewer enforcement stops than a two-man car, and drinking drivers pose special accident prevention problems from 2200 to 0230 hours. For these reasons and also for safety, the CHP has a state-wide policy which requires that cars will carry two men during hours of darkness. An exception to this policy may be granted by the zone commander which allows single-man units to be operated until 2200 hours.

FIGURE 1

MONTEREY COUNTY ACCIDENT RATE
BY DAY OF WEEK FOR 1971

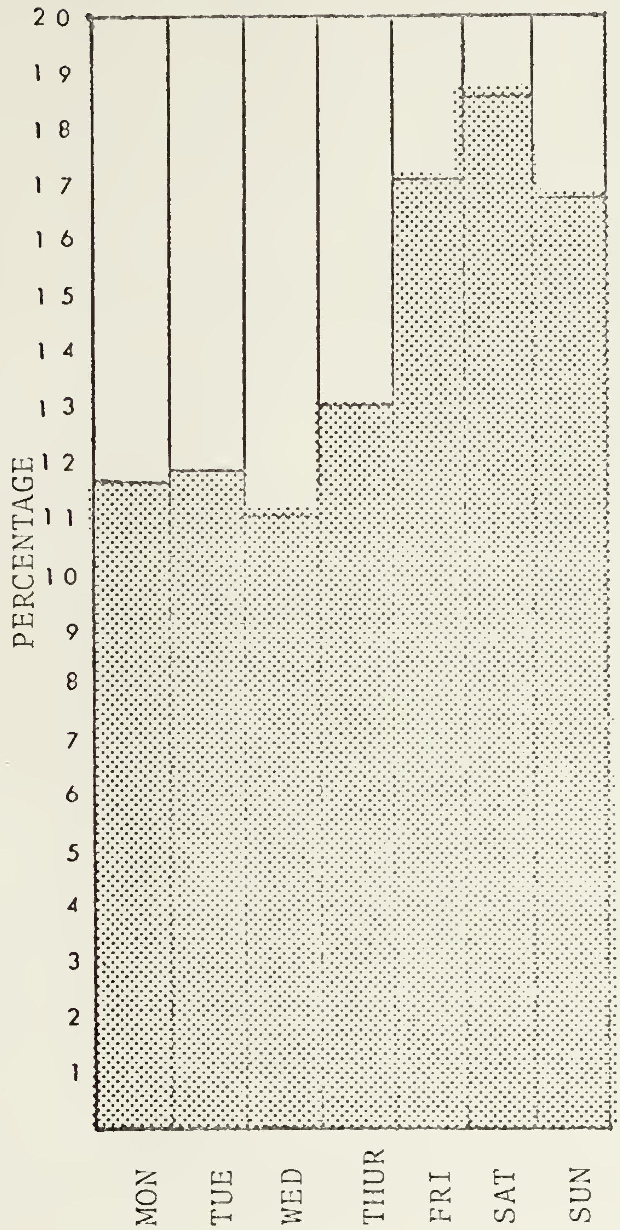
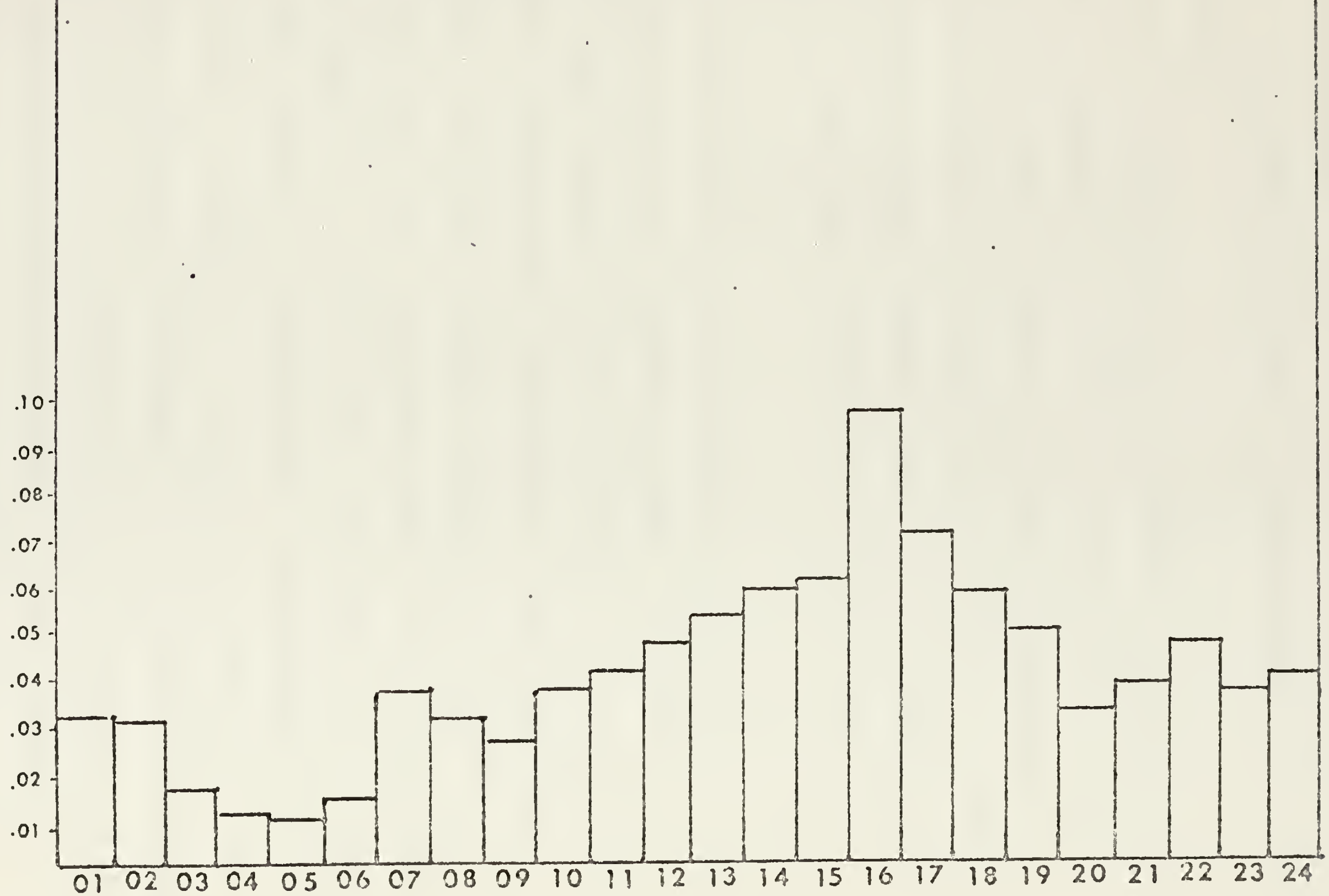


FIGURE 2

MONTEREY COUNTY ACCIDENT RATE
BY HOUR OF DAY FOR 1971



To model this manpower allocation problem as described above, it was necessary to make various assumptions. The relevant constraints which were quantified and used to formulate the problem precisely are discussed in the following paragraphs.

Because the distribution of accidents that governs the allocation of manpower is given in percentages, it was decided to model the problem in a form that would give required percentages of resources as a solution. This would also allow more general results that are applicable to a variety of combinations of manpower and cars that are available.

Due to the nature of the manpower constraints, it can be seen that there will always be total utilization of the number of men scheduled for any given day. The same statement cannot be made about the number of cars available. If, in fact, the number of men scheduled for each hour of the day does not equal the total number of cars available, the remaining cars will sit idle. With the above in mind, it was then decided to formulate the requirements in terms of percentages of manpower. It should be noted that once the manpower solution is obtained, it can then be translated into the number of men required which in turn has a direct linear relationship with the number of cars required. This is true even when considering the two-man cars required after dark.

With one-man cars patrolling during daylight and two-man cars patrolling after dark, it is assumed a two-man car has the same effectiveness as a one-man car. With this assumption, each man that is placed in a two-man car is half as effective in matching the accident distribution as each man in a one-man car. Because the model uses manpower requirements, the following transformation was made to maintain consistency between the distribution of accidents over the day and the required manpower allocation. The percentages of accidents during the hours of the day that required two-man cars on duty were doubled. In essence this means that for each increment in accident rate after dark, the manpower allocation is twice that required during daylight. The resulting curve was then renormalized to cause the percentages of accidents over the day to sum to unity.

Figure 3 shows what the resulting accident distribution becomes when the hour of darkness double-up period is set at 2000 hours. Figure 4 shows the same curve when the 2200 hours mandatory double-up policy is enforced.

To simplify the model, the 8-1/2 hour shifts were considered to be eight hours long since one-half hour is spent at the station and not patrolling. Also, each day of the week was assumed to have the same distribution of accidents. A more detailed discussion of this assumption is made in the next section.

FIGURE 3

DISTRIBUTION OF ACCIDENTS WITH
DOUBLE WEIGHT BETWEEN 2000 AND 0500

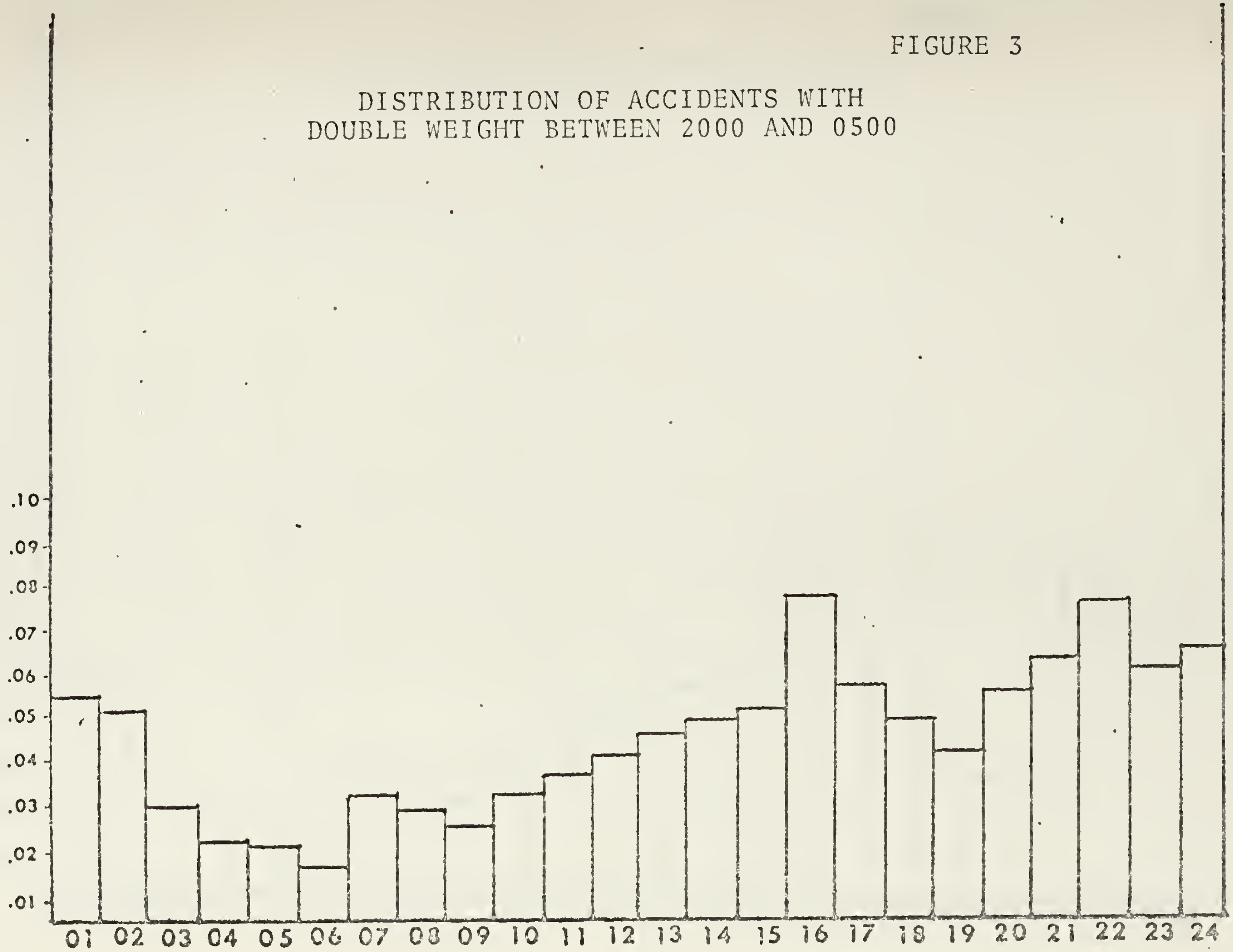
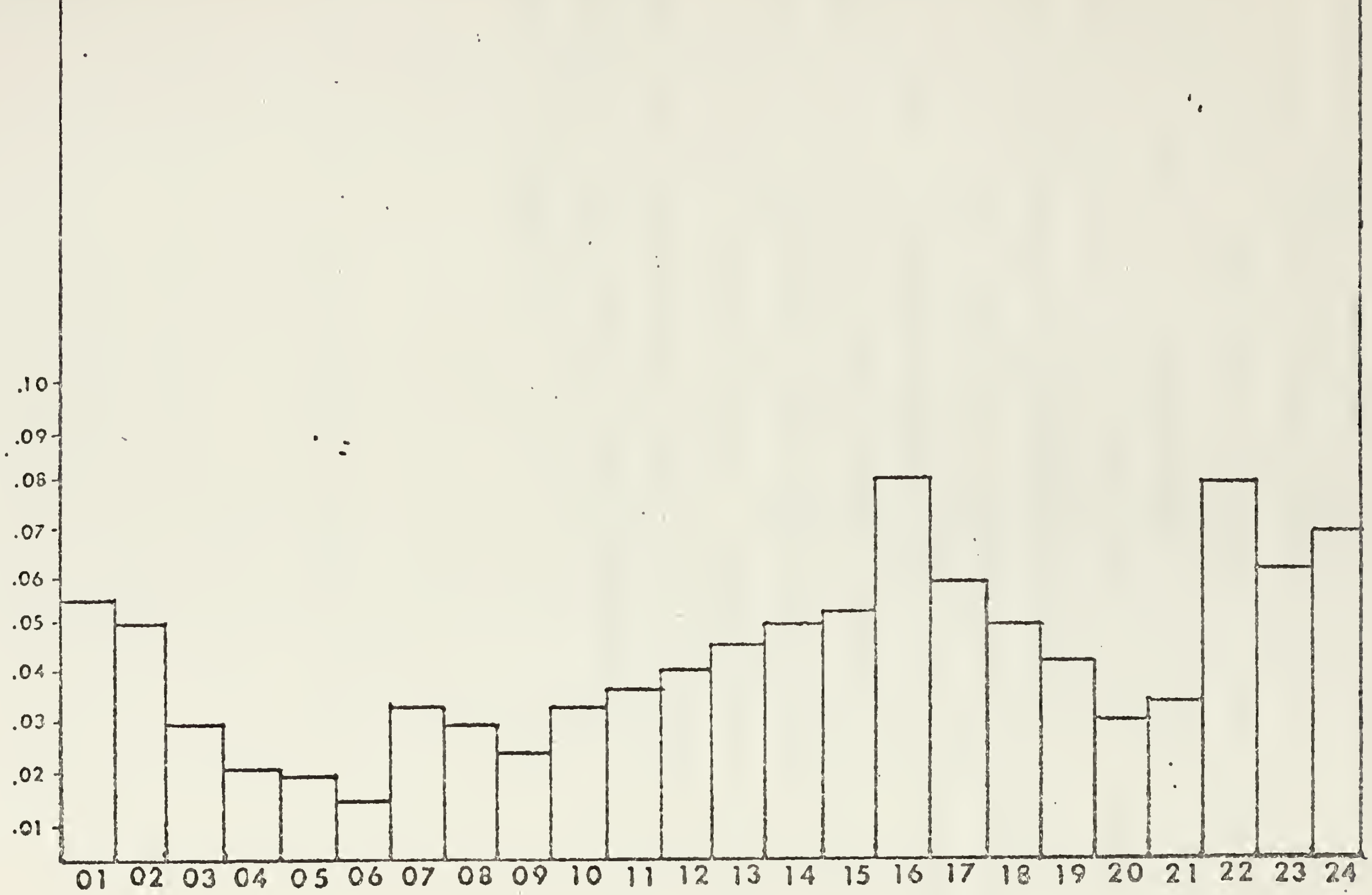


FIGURE 4

DISTRIBUTION OF ACCIDENTS WITH
DOUBLE WEIGHT BETWEEN 2200 and 0500



B. DATA ANALYSIS

The data analysis, as stated earlier, was a secondary benefit which evolved from examining the assumption that each day of the week had the same distribution of accidents by hour of the day. Basically, in order to model the problem over a one-day period, it was necessary to determine if there was a correlation between the total number of accidents by day and the distribution of accidents by time of day.

The Monterey Area maintains a three-by-five index card on each accident. The data on the cards were transferred to computer cards in the format shown in Figure 5. The distribution for each day of the week was then plotted against the overall accident rate. Although no statistical tests were run, the trends of each day approximated the overall accident rate curve. These comparisons are shown in Figures 6 through 12.

FIGURE 5

CARD FORMAT FOR 1971 DATA

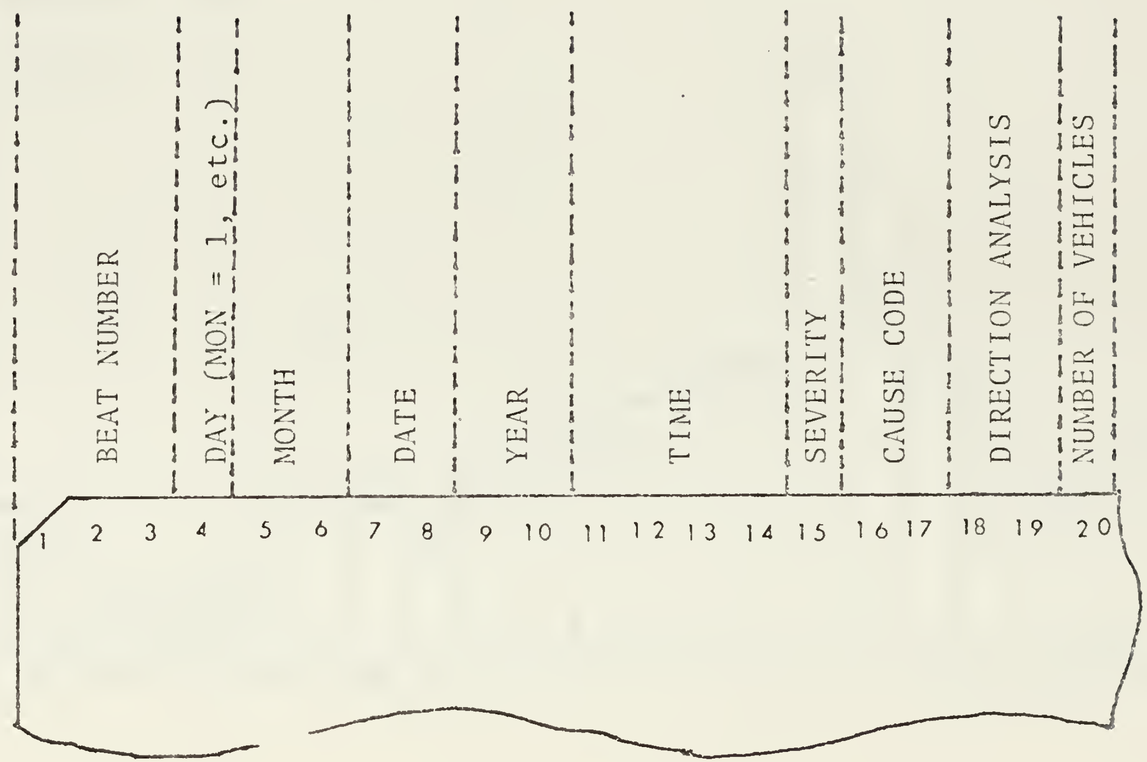
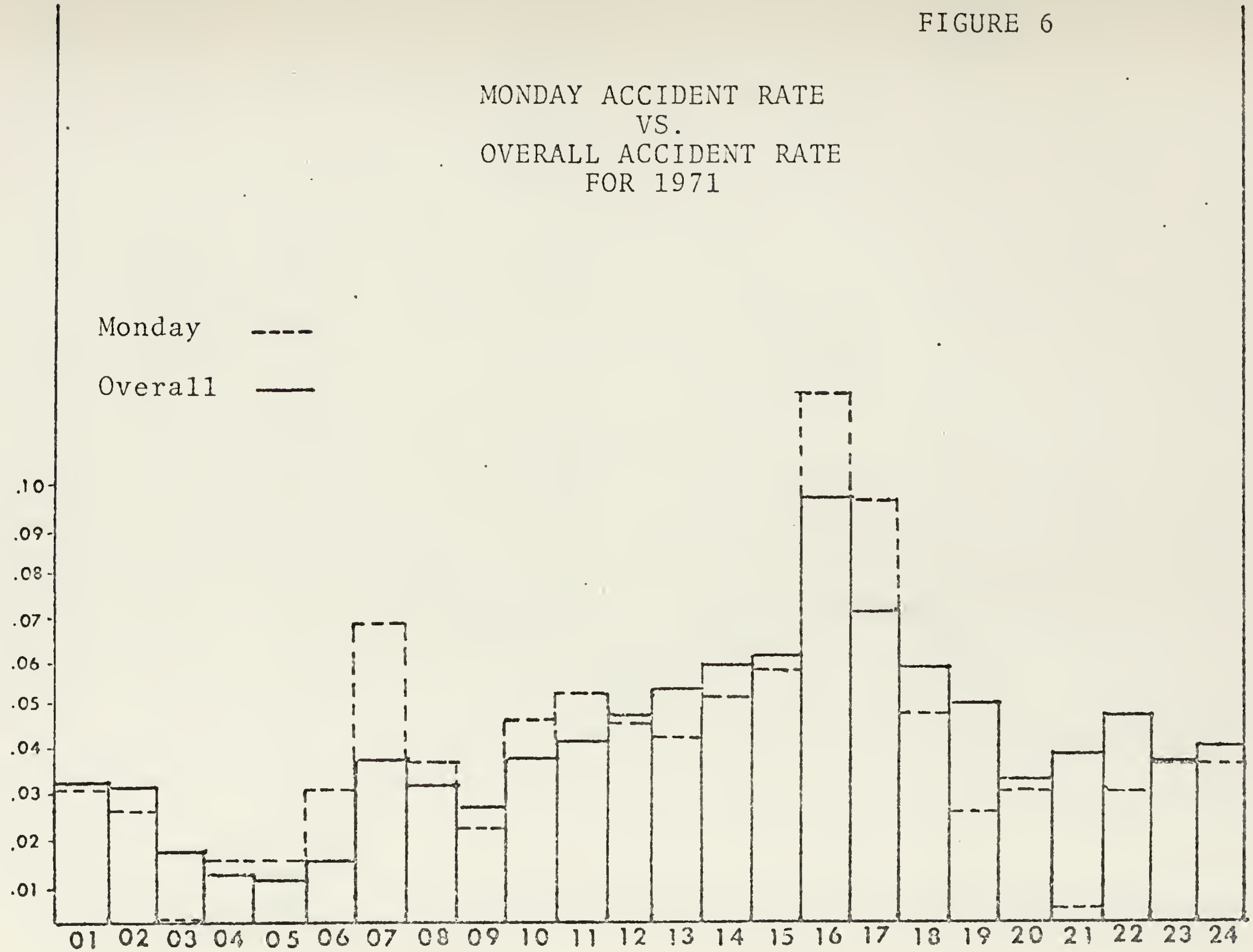


FIGURE 6

MONDAY ACCIDENT RATE
VS.
OVERALL ACCIDENT RATE
FOR 1971



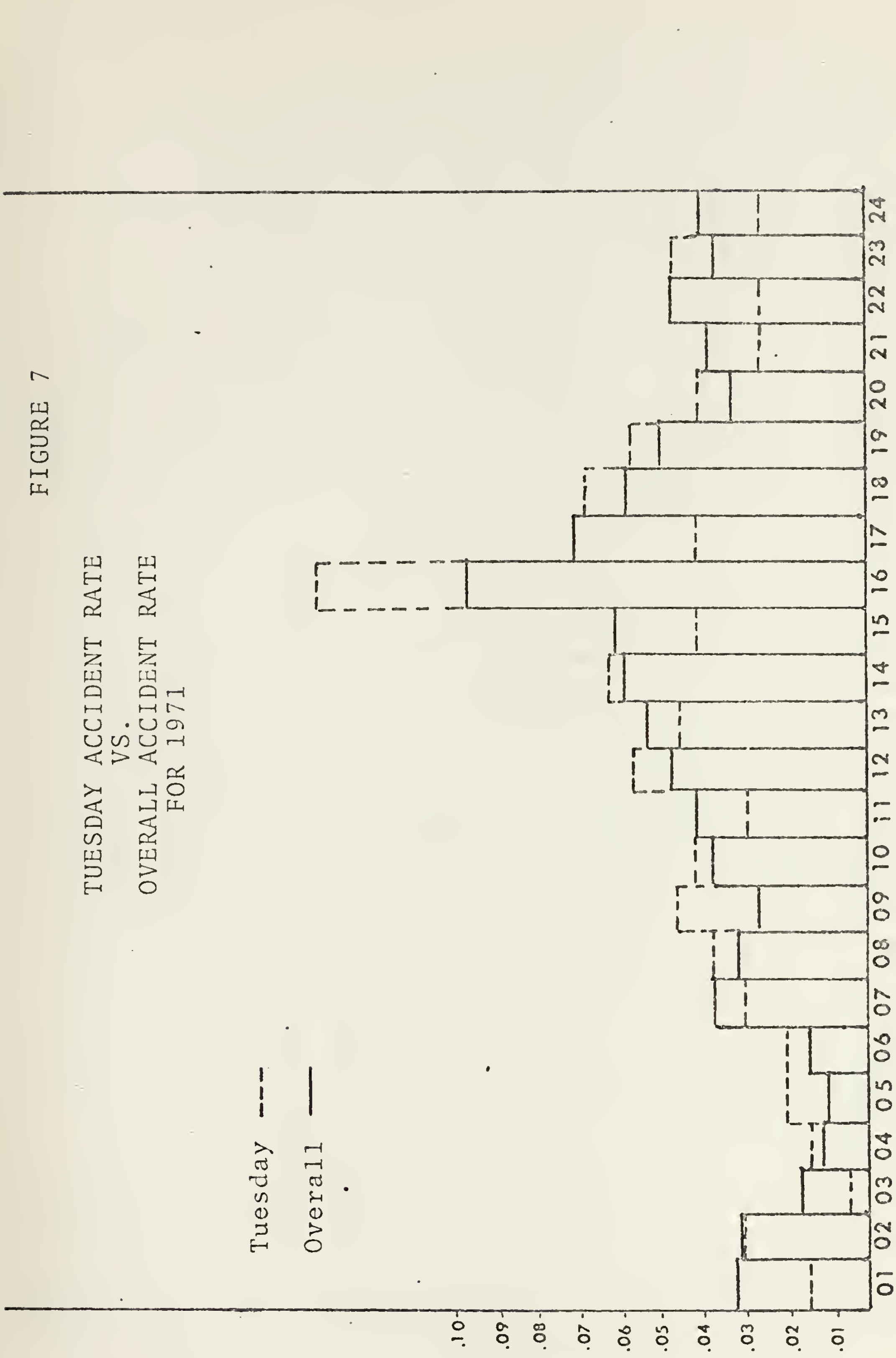


FIGURE 7

TUESDAY ACCIDENT RATE
VS.
OVERALL ACCIDENT RATE
FOR 1971

FIGURE 8

WEDNESDAY ACCIDENT RATE
VS.
OVERALL ACCIDENT RATE
FOR 1971

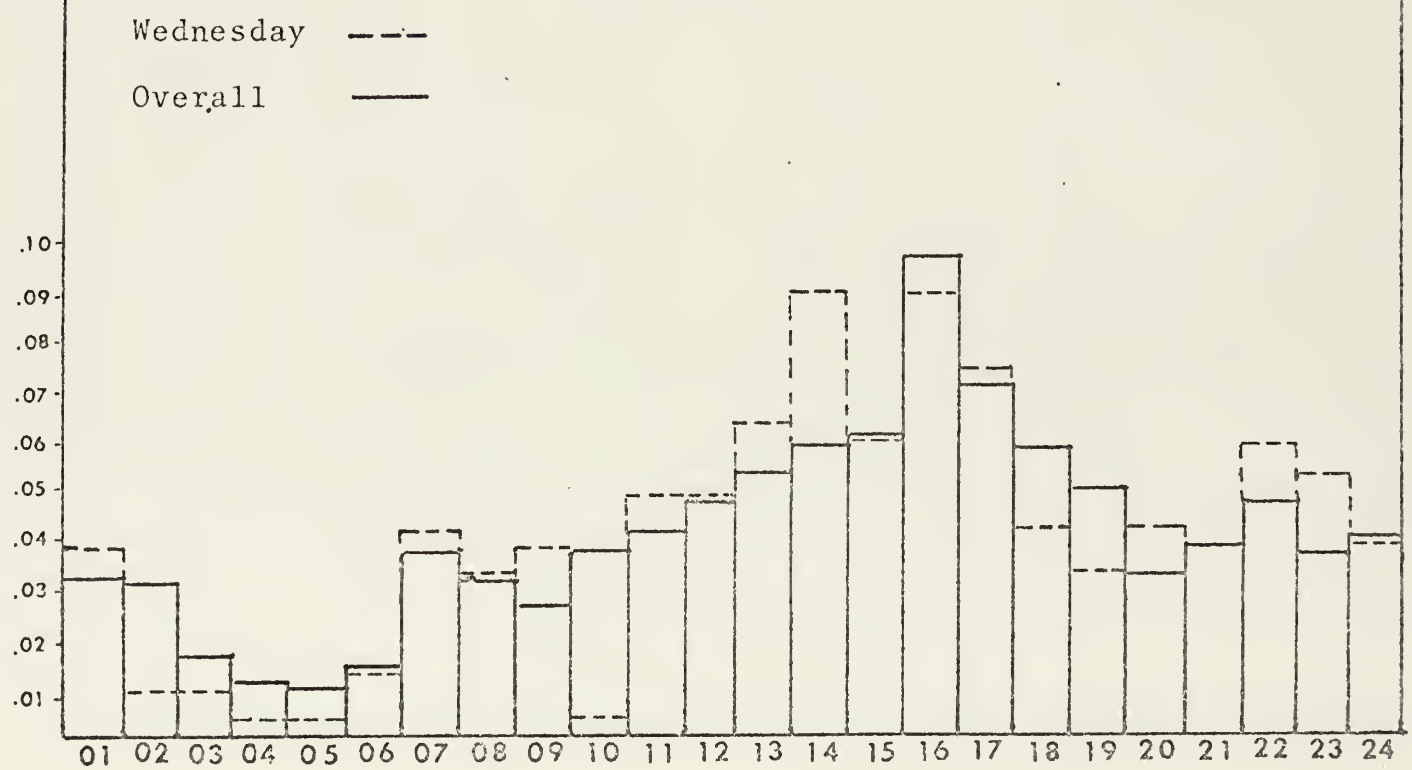


FIGURE 9

THURSDAY ACCIDENT RATE
VS.
OVERALL ACCIDENT RATE
FOR 1971

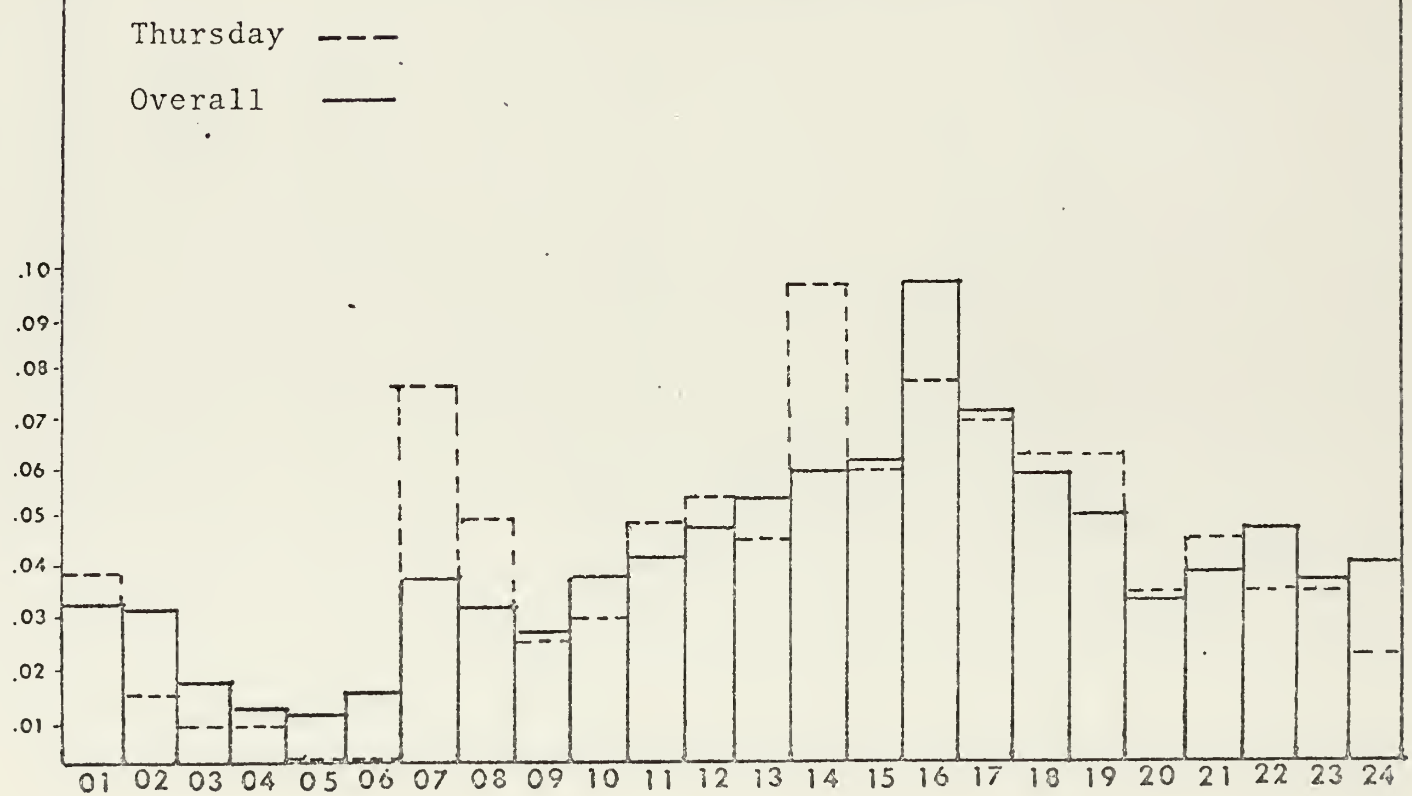


FIGURE 10

FRIDAY ACCIDENT RATE
VS.
OVERALL ACCIDENT RATE
FOR 1971

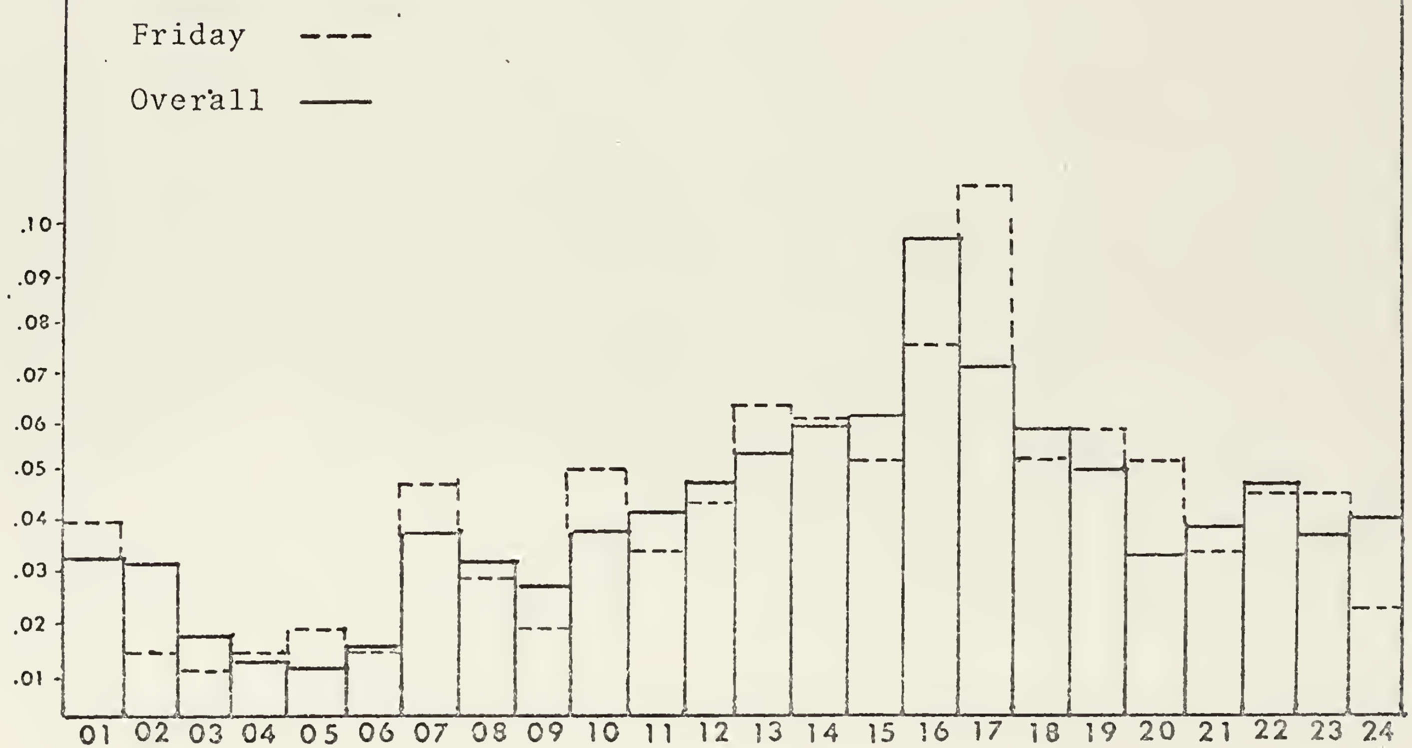


FIGURE 11

SATURDAY ACCIDENT RATE
VS.
OVERALL ACCIDENT RATE
FOR 1971

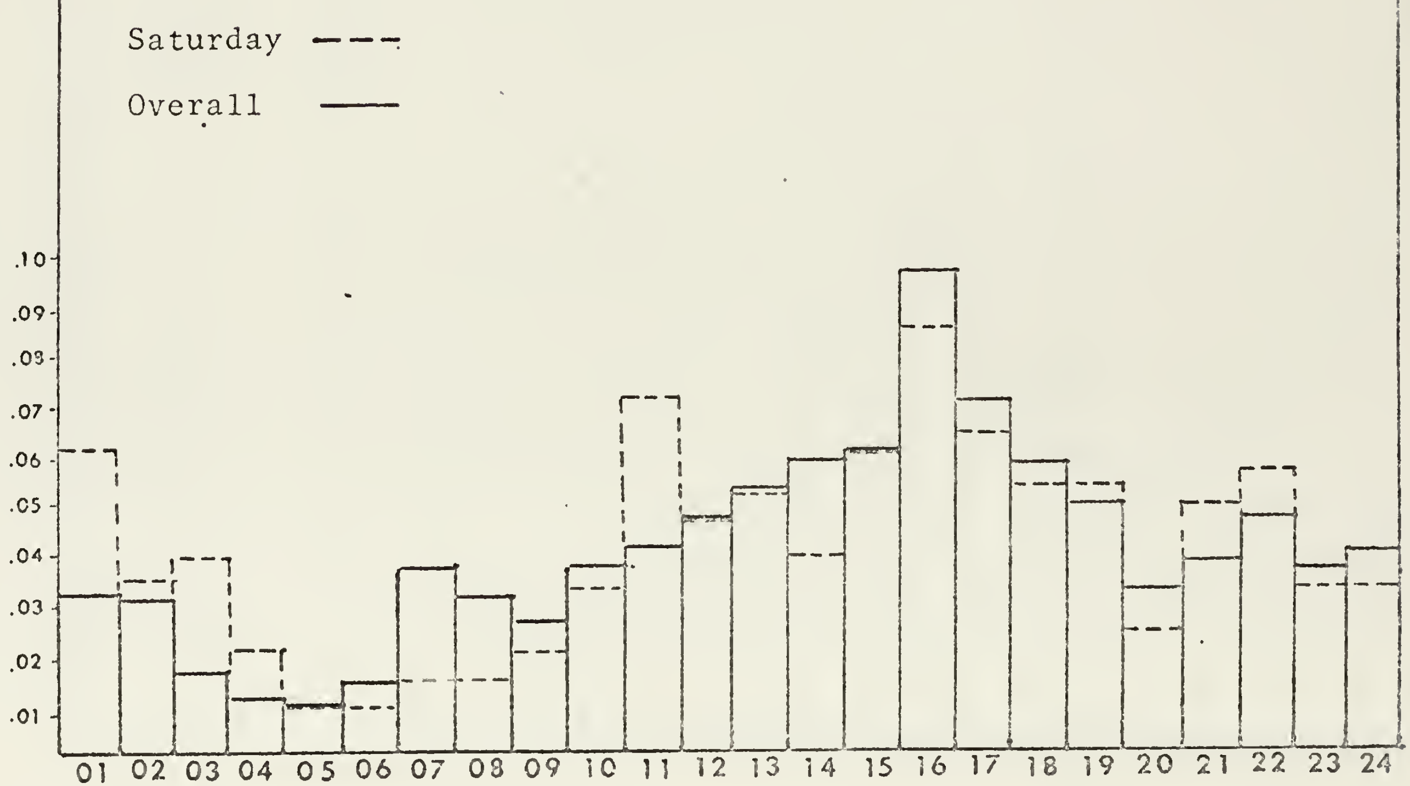
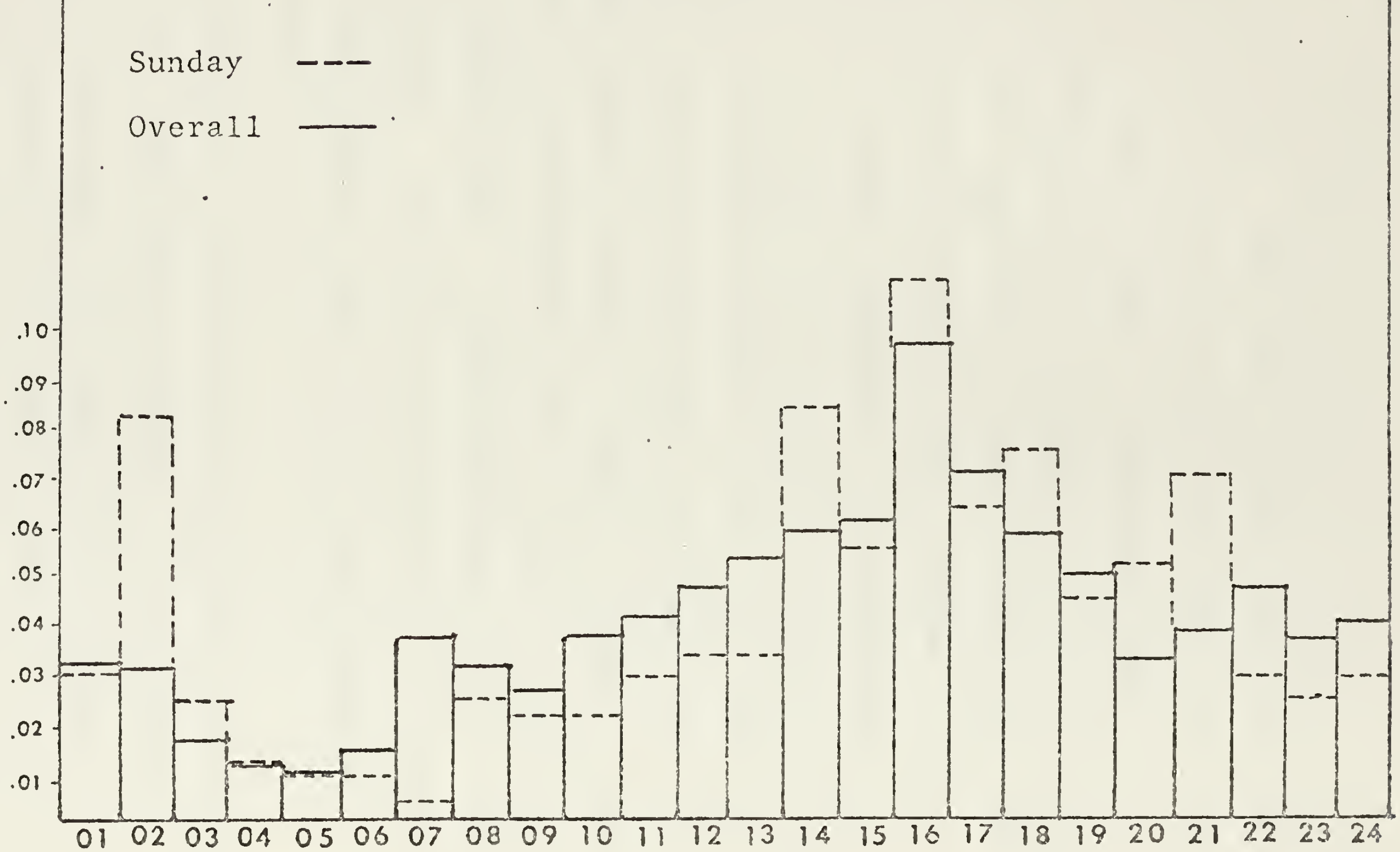


FIGURE 12

SUNDAY ACCIDENT RATE
VS.
OVERALL ACCIDENT RATE



III. FORMULATION

A. MODEL SELECTION

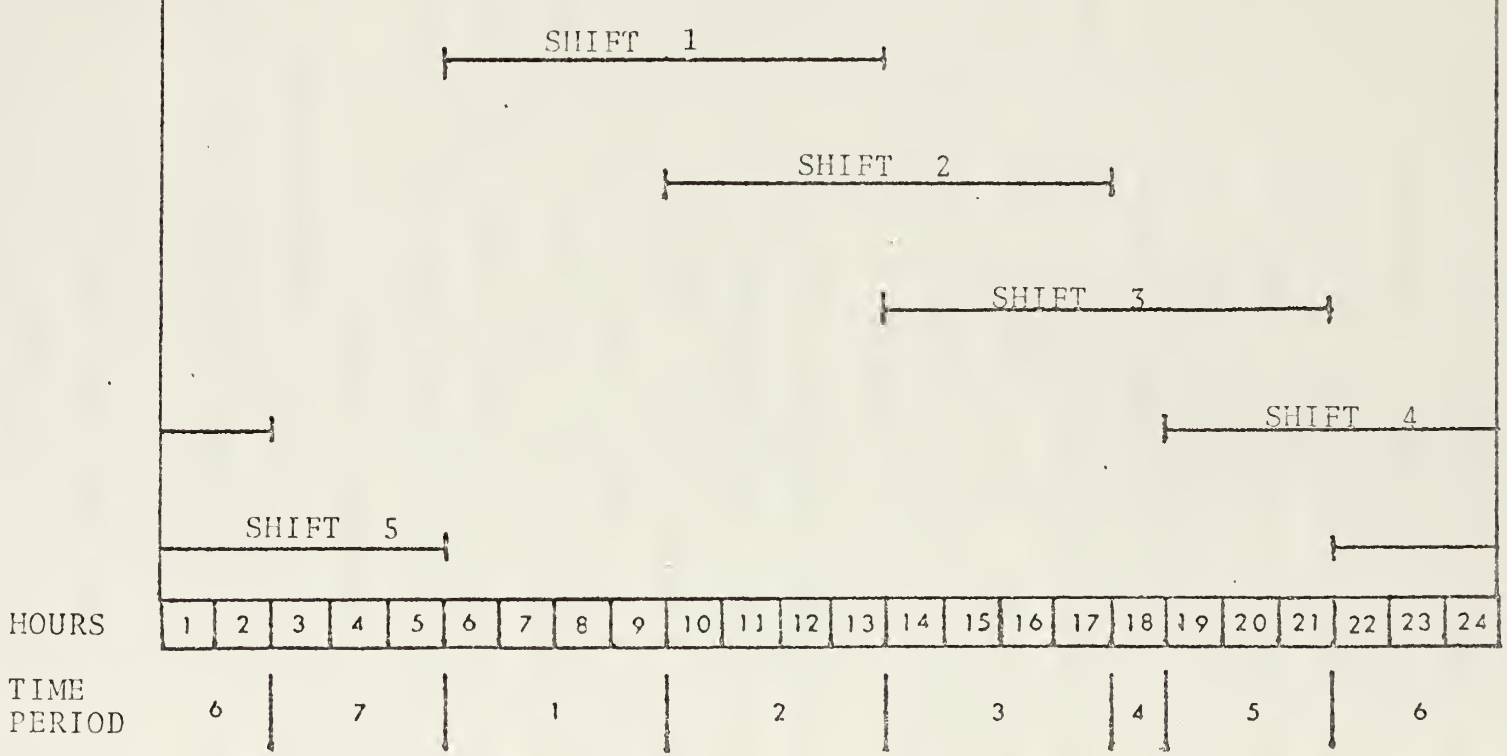
Prior to formulating a model, a close examination of the constraints was made. When taken singly, the constraints were observed to be linear and this initially led to the choosing of some form of linear programming model.

Before proceeding with the formulation, it is necessary to define the terms which will be used in developing the model. The objective function in this problem is the benefit from the match of manpower to the accident rate. The benefit is in terms of percentage of manpower allocation and therefore its maximum value is unity. Let the term "time period" represent those hours of the day when the shifts on duty remain constant. Figure 13 shows the shifts that were used by the Monterey Area during 1971 and their associated time periods. A time period may contain from one to eight hours. Let Y_i be the percentage of manpower assigned to time period i and X_j be the percentage of manpower assigned to shift j.

The following is a discussion of the objective function. The benefit in a time period increases as the manpower assigned to it increases. The rate of the increase in benefit is dependent upon the number of hours in the time period. A useful analogy to benefit is the area under the accident rate curve. If a time period extends over four hours of the day, a unit increase in manpower benefits all

FIGURE 13

1971 SHIFT PERIODS FOR MONTEREY COUNTY



four hours and therefore the area is increasing at a rate of four to one. It is at this point that the linearity breaks down.

To better describe the nonlinearity of the objective function, it is advantageous to utilize a graphical description. Figure 14 depicts a sample time period consisting of four hours. The percentage of accidents is plotted on the vertical axis and the hours are plotted on the horizontal axis. As the percentage of manpower assigned to the time period increases along the vertical axis, the benefit increases with a rate of four to one. This rate of increase is depicted as a slope in Figure 15 which has benefit, b, plotted as a function of Y.

The slope of the benefit function remains constant until Y reaches the .07 manpower level. At this point any further increase in Y causes excess manpower in the first hour and is considered to have no additional benefit. However, the remaining three hours still accumulate benefit. It can be seen from Figure 15 that the slope of the benefit function decreases each time the percentage of manpower exceeds the value of the accident percentage for the next lowest hour. When Y reaches .15, any additional increase in manpower will give zero additional benefit to the time period.

The benefit derived from each time period is a piecewise linear function of Y, denoted by B(Y) = b. The model can now be written as:

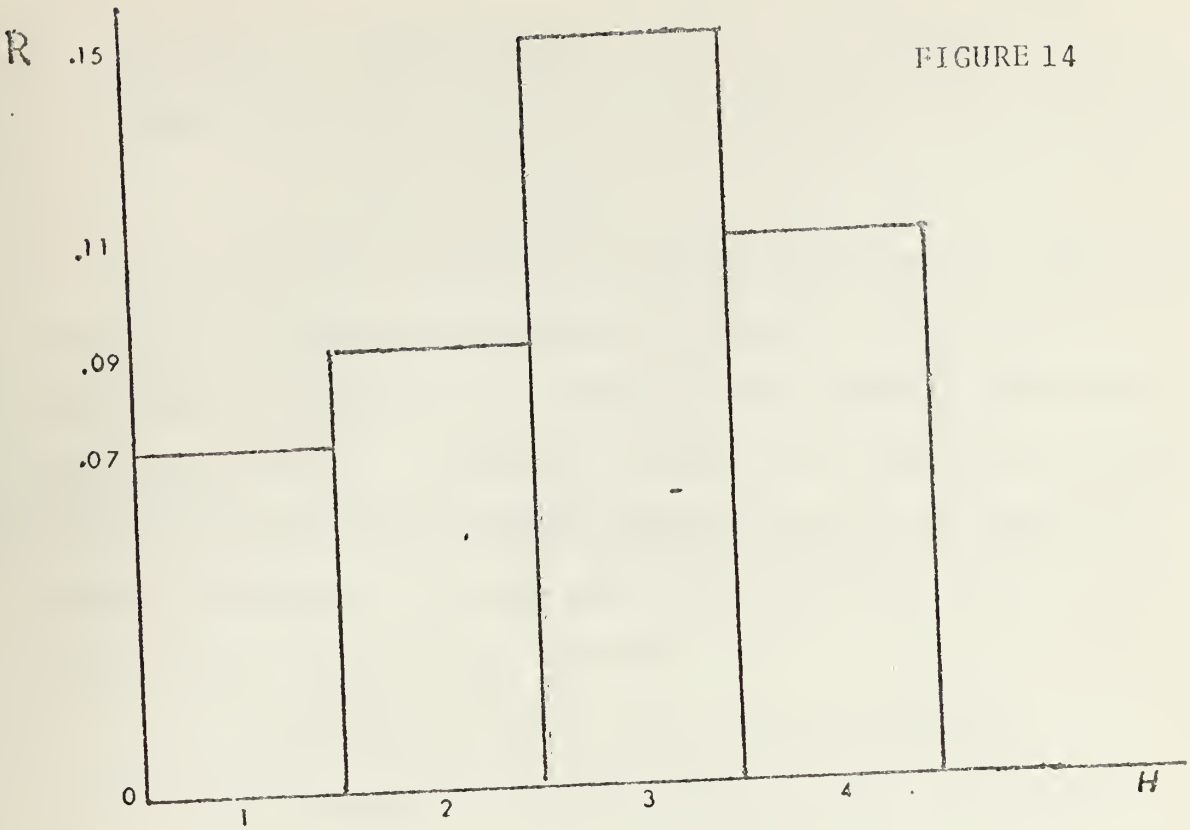


FIGURE 14

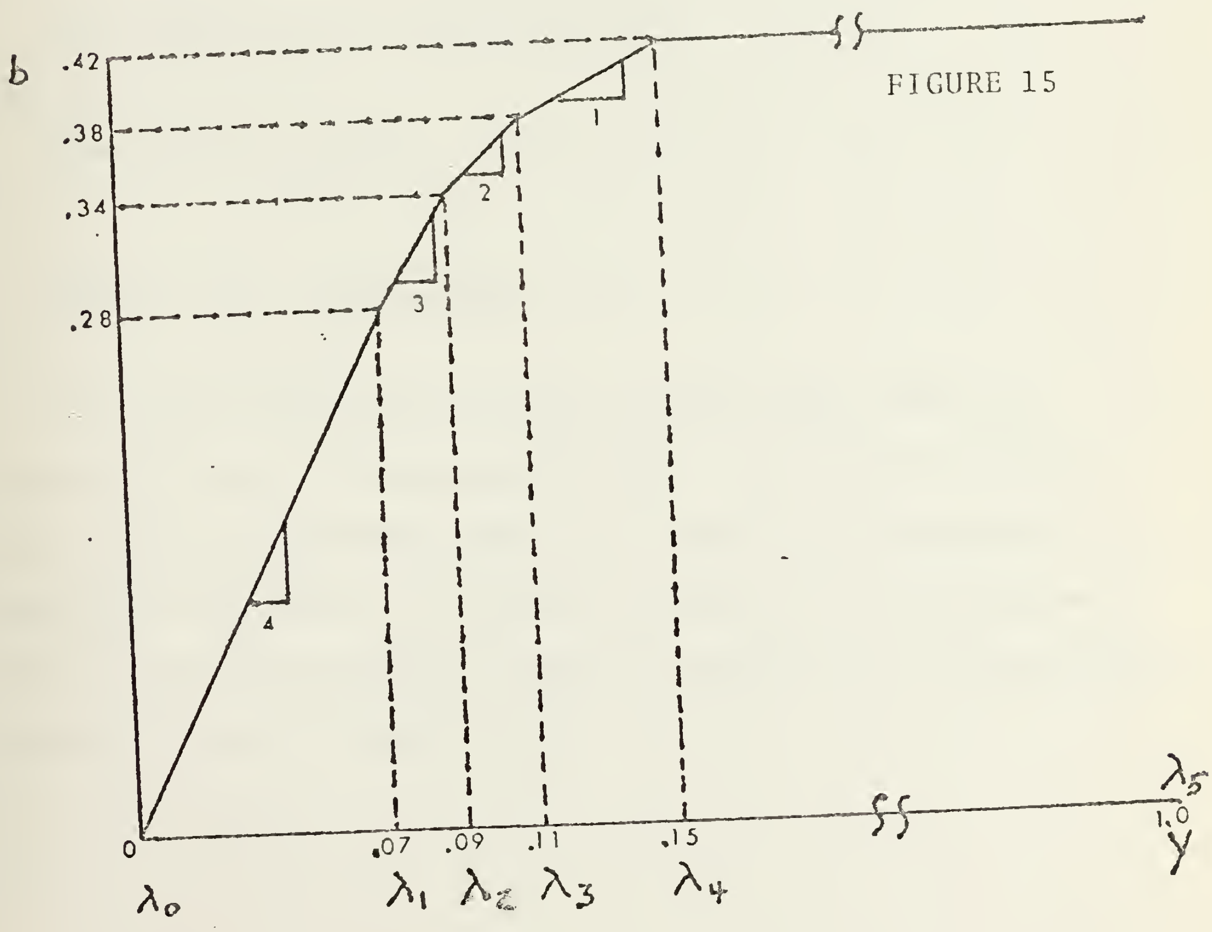


FIGURE 15

maximize: Benefit (Y)

subject to: The constraints on the X's

Where $Y = f(X)$.

Let Y_i be the manpower allotted time period i, $(i=1...k)$, and X_j be the manpower allotted to shift j, $(j=1...m)$. Y is now a linear function of X which is described by the matrix multiplication $Y = AX$ where Y is the $k \times 1$ vector of manpower assigned to the time periods and X is the $m \times 1$ vector of manpower assigned to the shifts. The matrix A is a $k \times m$ matrix whose elements a_{ij} satisfy

$$a_{ij} = \begin{cases} 1 \text{ if shift } j \text{ is on duty in time period } i \\ 0 \text{ if shift } j \text{ is not on duty in time period } i. \end{cases}$$

The model can be rewritten as:

$$\text{maximize:} \quad \sum_{i=1}^{k} B(Y_k)$$

$$\text{subject to:} \quad \sum_{i=1}^{k} Y_i - [A]X_j = 0.$$

The next step was to write the nonlinear model in separable form. To accomplish this, the "Lambda Method" was used as presented in Ref. 3. Using this method the additional variable λ is used to indicate the intervals of the separable function as shown on Figure 15. The resulting problem is written as follows:

maximize: $$\sum_{i=1}^{k} \sum_{j=1}^{N_k} b_{ij}\lambda_{ij}$$

subject to: $$\sum_{j=1}^{N_k} Y_j\lambda_{ij} - f(X) = 0 \qquad \forall \ i=1,\ldots k$$

$$\sum_{j=1}^{N_k} \lambda_{ij} = 1 \qquad \forall \ i=1,\ldots k$$

$$\sum_{\ell=1}^{M} X_\ell = .125$$

$$\lambda_{ij} \geq 0 \ \forall \lambda_{ij}$$

$$X_\ell \geq 0 \ \forall X_\ell \ .$$

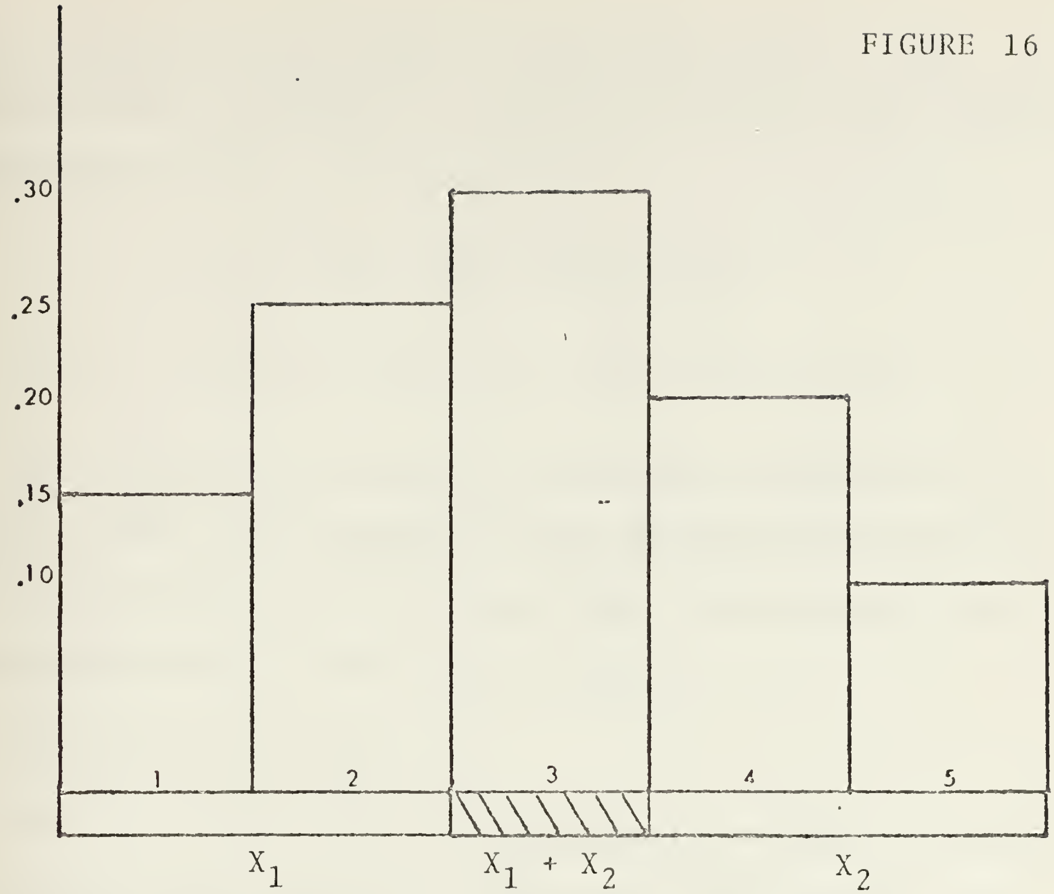
K equals the number of time periods per day.

N_k equals the number of λ's per time period.

M equals the number of shifts to be considered.

The value of the sum of the X's in the above model needs to be explained further. Intuitively it can be reasoned that since each shift will contain a percentage of the total manpower, the total sum of the X's, which relates to the percentage assigned, should equal unity. The difficulty occurs when considering the units of the Y's and the X's. Again the point can best be explained graphically. Figure 16 depicts a hypothetical five-hour day with two three-hour shifts that overlap during the third hour. As can be seen from the computations, the total allocation of manpower over each hour of the day must sum to unity. From this the sum of $X_1 + X_2$ must equal 1/3 or

FIGURE 16



X_1 = % MEN ON SHIFT 1

X_2 = % MEN ON SHIFT 2

THEREFORE:

HOUR	1	2	3	4	5

$$\% \quad X_1 + X_1 + [X_1 + X_2] + X_2 + X_2 = 1$$

$$3X_1 + 3X_2 = 1$$

$$X_1 + X_2 = 1/3$$

unity divided by the number of hours per shift. This concept can be enlarged to encompass a 24-hour day with five eight-hour shifts. It now becomes:

$$8_{X_1} + 8_{X_2} + 8_{X_3} + 8_{X_4} + 8_{X_5} = 1$$

or $$X_1 + X_2 + X_3 + X_4 + X_5 = 1/8 = .125.$$

For illustrative purposes the model is written out in complete form (See Appendix B) for the 1971 starting times used by the Monterey Area of the CHP. The accident rate distribution used is the one in Figure 3.

B. COMPUTER UTILIZATION

Because of the extensive manipulation of data required, the computer was utilized to formulate the separable programming model for each set of starting times examined. A program was written (not included in this thesis) in Fortran IV which, when given the starting times and the constraining distribution of accidents as data, punched th data on cards in a format adaptable to the MPS/360. The allocation of manpower per shift was then computed. These values were then read into a third program which plotted the manpower allocation curves superimposed on the accident rate curves.

C. DATA ANALYSIS

The technique employed in the analysis of the data was to examine graphically the accident trends over the various

areas of the county. Plots of the various areas were made in an attempt to identify specific accident trends related to day of the week and time of the day. The results of these plots are discussed in the summary.

IV. SUMMARY

A. RESULTS OF MANPOWER ALLOCATION

Upon completion of the model, the next step was to obtain useful results. The approach taken was not to arrive at a single solution, but rather a set of alternative solutions from which comparisons could be made. Seventeen sets of starting times and two accident rate curves were chosen for comparison. (See Table I and Table II.)

One measure of effectiveness which could be used for comparison is the value of the objective function for each solution. The maximum value possible is unity, but it is not attained because of the nature of the constraints. The value of the objective function gives relative comparisons. If more information is needed, a second method may be used. This method plots the manpower distribution superimposed on the accident rate versus time of day, thereby allowing a visual comparison of the manpower and accident rate (Appendix C and Appendix D). An interesting comparison to make at this point is between the allocation of manpower used in 1971 by the Monterey Area and the solution obtained from this model. (See Figures 17 and 18.) This gives an indication of the effectiveness of the model in assigning manpower for a problem of this nature.

There is a third measure which must also be considered. After choosing a set of starting times, the number of patrol

TABLE I

SHIFT STARTING TIMES VS. 2000 DOUBLE-UP POLICY

SHIFT STARTING TIMES 1, 2, 3, 4, 5	PER CENT PER SHIFT 1	2	3	4	5	OBJECTIVE FUNCTION
06, 10, 12, 19, 22	.168	.064	.312	.320	.136	.875
06, 10, 14, 18, 22	.160	.136	.248	.168	.228	.903
06, 10, 14, 19, 22 *	.168	.128	.256	.224	.224	.910
06, 10, 13, 18, 22	.160	.136	.248	.168	.288	.876
06, 10, 13, 20, 22	.184	.080	.304	.288	.144	.890
06, 10, 12, 18, 22	.112	.152	.232	.360	.144	.861
06, 10, 12, 20, 22	.168	.064	.312	.320	.136	.884
05, 10, 15, 22, 22 +	.168	.128	.360	.000	.344	.851
06, 14, 14, 19, 22 **	.232	.000	.360	.200	.208	.891
06, 12, 12, 19, 22 **	.184	.000	.360	.312	.144	.867
05, 10, 14, 21, 21 +	.144	.144	.304	.000	.408	.874
06, 12, 12, 20, 22 **	.184	.000	.360	.416	.040	.876
06, 11, 11, 19, 22 **	.136	.000	.360	.360	.144	.879
06, 13, 13, 19, 22 **	.232	.000	.360	.200	.208	.868
06, 10, 13, 19, 22	.168	.096	.288	.240	.208	.884
06, 10, 14, 20, 22	.168	.120	.304	.176	.232	.909
06, 14, 14, 20, 22 **	.232	.000	.360	.264	.144	.891

* 1971 Shift Times
\+ Eliminates Shift 4
** Eliminates Shift 2

TABLE II

SHIFT STARTING TIMES VS. 2200 DOUBLE-UP POLICY

SHIFT STARTING TIMES	PER CENT PER SHIFT					OBJECTIVE
1, 2, 3, 4, 5	1	2	3	4	5	FUNCTION
06, 10, 12, 19, 22	.176	.072	.320	.224	.208	.885
06, 10, 14, 18, 22	.136	.176	.208	.120	.360	.886
06, 10, 14, 19, 22 *	.176	.136	.264	.064	.360	.882
06, 10, 13, 18, 22	.120	.192	.208	.176	.304	.881
06, 10, 13, 20, 22	.176	.104	.304	.256	.160	.883
06, 10, 12, 18, 22	.096	.184	.200	.224	.296	.881
06, 10, 12, 20, 22	.176	.072	.328	.256	.168	.891
05, 10, 15, 22, 22 +	.136	.208	.256	.000	.400	.854
06, 14, 14, 19, 22 **	.248	.000	.352	.000	.400	.853
06, 12, 12, 19, 22 **	.176	.000	.384	.224	.216	.876
05, 10, 14, 21, 21 +	.144	.168	.256	.000	.432	.852
06, 12, 12, 20, 22 **	.184	.000	.384	.256	.176	.882
06, 11, 11, 19, 22 **	.136	.000	.384	.320	.160	.880
06, 13, 13, 19, 22 **	.216	.000	.352	.208	.224	.860
06, 10, 13, 19, 22	.176	.104	.288	.208	.224	.881
06, 10, 14, 20, 22	.176	.104	.296	.000	.424	.874
06, 14, 14, 20, 22 **	.248	.000	.352	.000	.400	.853

* 1971 Shift Times
\+ Eliminates Shift 4
** Eliminates Shift 2

FIGURE 17

MODEL SOLUTION FOR
MANPOWER ALLOCATION VS. ACCIDENT RATE
1971

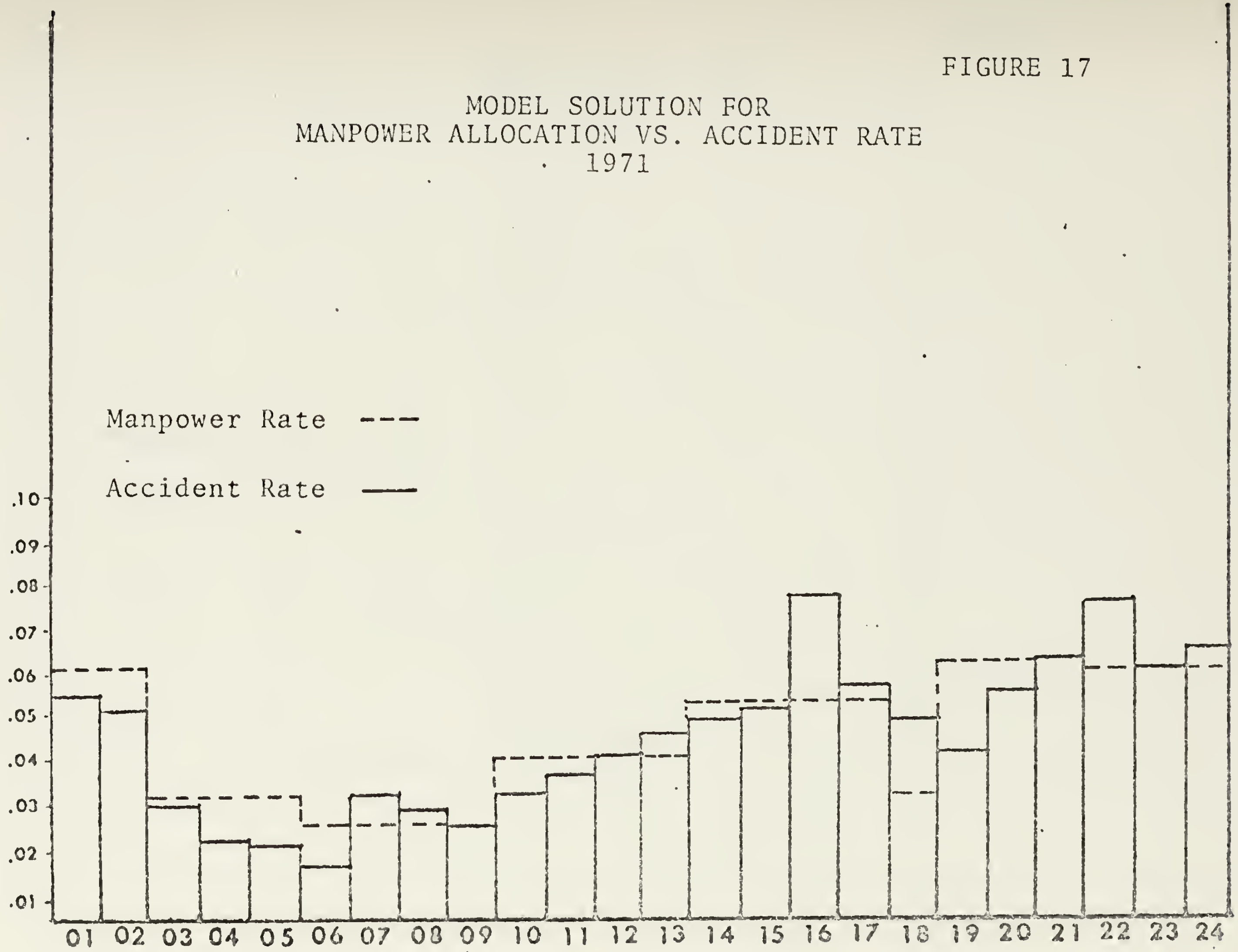
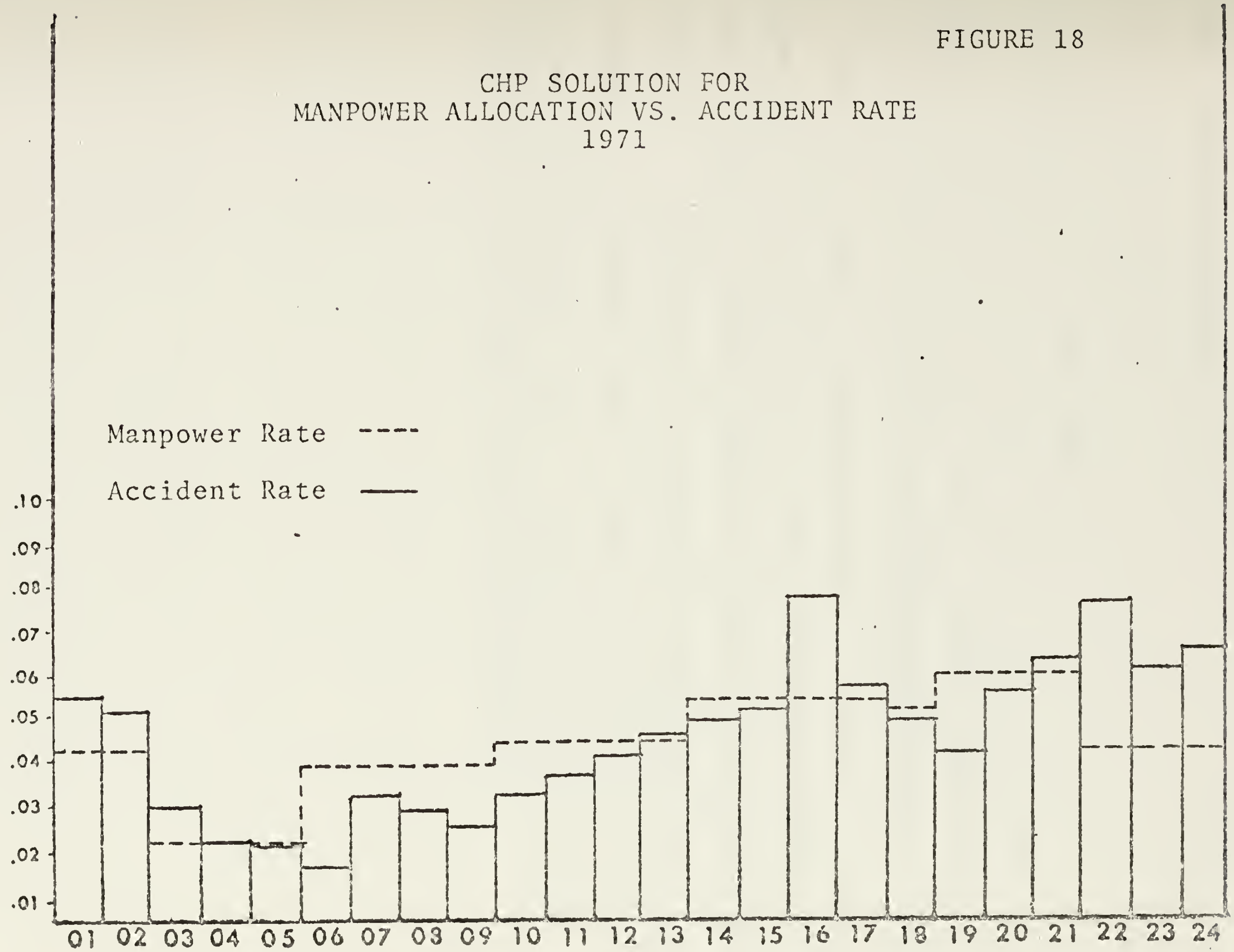


FIGURE 18

CHP SOLUTION FOR
MANPOWER ALLOCATION VS. ACCIDENT RATE
1971



cars required to meet the implied manpower allotments must be computed. This effort is within the capability of the local areas and includes consideration of the numbers of men and cars available.

B. RESULTS OF THE DATA ANALYSIS

By examining the accident distribution plots for various areas of Monterey County, a better description of the specific trends throughout the county was obtained. Because of the large number, the plots were not included in this paper but were given to the Monterey Area CHP for their use. A note of caution should be given as to the use of this data. Because of the relatively small data base of one year, any trends must be used only as rough guidelines for decision making. This is due to the unknown parent distribution from which the data was obtained.

V. CONCLUSIONS

The method developed in this thesis is considered to be a valid starting point in solving the overall problem of manpower allocation. The results obtained thus far have immediate application, giving the CHP a reasonable alternative to their present method. However, there are many other questions that remain unanswered and should be addressed further.

As mentioned in the introduction, allocation of manpower is made from considering the distribution of accidents from the previous six to 12 months. No attempt was made to fully describe the distribution of accidents, thereby establishing a fixed distribution and the pertinent parameters. If this were accomplished, the model could be re-employed to obtain results which would be applicable for a long period of time and not be dependent upon the previous year's distribution. This approach could then be further implemented to describe differences in the accident distributions throughout various beats in any area. The next logical step would then be to construct a model for deployment of officers over the various beats.

As for the model itself, it was employed in a simple manner in solving the problem of manpower allocation. It is recognized that, to solve the model, access to a computer facility is necessary. However, it should also be recognized

that the model can be applied to the CHP problem or any other similar type problem in a myriad of ways. As an example, one could enlarge the time period from a day to a week in order to determine a manning schedule. This concept could be extended to cover a period of a month or even a year to assist in determining optimal times for vacations. Also, another variation would be to establish upper and lower bounds on the number of men required.

APPENDIX A

PRE-LIEUTENANT'S CLASS
PERSONNEL DEPLOYMENT

1. There are approximately 1784 annual man-hours available to you for scheduling per man. This considers vacation, sick leave, etc.

2. Divide 1784 by the 52 weeks in the year and we find that each man can provide us with approximately 34 man-hours per week.

3. To determine the number of weekly man-hours available to us for scheduling, we multiply 34 man-hours times the number of men available to us for traffic enforcement. In other words, if we have a 44-man squad we will assume that after special detail and the like are considered, there will be 40 men available to us for traffic supervision purposes during the month. Thirty-four multiplied by 40 provides the total of man-hours available to us for each week of the month, or 1360 man-hours.

4. In reviewing our Deployment Control Reports, we find that in the last six months, or during a similar season last year, our particular area experienced:

12.8%	of its	total	accidents	on	Mondays
12.3%	"	"	"	"	Tuesdays
11.9%	"	"	"	"	Wednesdays
12.4%	"	"	"	"	Thursdays
16.6%	"	"	"	"	Fridays
18.0%	"	"	"	"	Saturdays
16.0%	"	"	"	"	Sundays

5. To determine the manpower assignments in proportion to the accident experience by day of week, we must multiply 1360 (weekly hours available) by the per cent of accidents each day of the week. Here is how it should look using the foregoing accident experience.

Mondays	- 12.8% of 1360	= 174	man-hours
Tuesdays	- 12.3% of 1360	= 167	man-hours
Wednesdays	- 11.9% of 1360	= 162	man-hours
Thursdays	- 12.4% of 1360	= 169	man-hours
Fridays	- 16.6% of 1360	= 226	man-hours
Saturdays	- 18.0% of 1360	= 245	man-hours
Sundays	- 16.0% of 1360	= 217	man-hours
TOTALS	100%	1360	man-hours

6. To convert these daily requirements into men we divide the man-hours required each day by eight hours (one man's shift) and come up with the number of men we should have on duty each day of the week to correspond with the accident experience.

Mondays	- 174 divided by 8	= 22 men	(nearest whole number)
Tuesdays	- 167 " " 8	= 21 men	(" " ")
Wednesdays	- 162 " " 8	= 20 men	(" " ")
Thursdays	- 169 " " 8	= 21 men	(" " ")
Fridays	- 226 " " 8	= 28 men	(" " ")
Saturdays	- 245 " " 8	= 31 men	(" " ")
Sundays	- 217 " " 8	= 26 men	(" " ")

7. The same principles can be applied in determining the number of men to assign to each respective shift. A time of day study over a six-month or longer period should provide a reasonably accurate guide to the accident pattern. For example, let us assume that such a study revealed that our shifts should be established, generally, as follows:

Shift #1 - 0001 - 0800 hours - 10% of the accidents
Shift #2 - 0700 - 1500 hours - 25% of the accidents
Shift #3 - 1400 - 2200 hours - 50% of the accidents
Shift #4 - 1600 - 2400 hours - 15% of the accidents

In applying the 40 available men to this pattern, we would assign 10% or four men to Shift #1, 25% or 10 men to Shift #2, 50% or 20 men to Shift #3, and 15% or six men to Shift #4.

Many other factors would normally enter the picture, such as vacations, fixed post assignments, injury time off, and the like, which would reduce the available manpower. However, the foregoing is provided as a guide and to offer a basis from which to begin.

APPENDIX B

APPLICATION OF SEPARABLE PROGRAMMING MODEL FOR SHIFT STARTING TIMES 06, 10, 14, 19, 22 AND FOR A 2000 TWO-MAN CAR POLICY

OBJECTIVE FUNCTION

maximize:

$$
\begin{aligned}
\Big\{ & \left[0\lambda_{01} + .048\lambda_{11} + .075\lambda_{21} + .082\lambda_{31} + .087\lambda_{41} + .087\lambda_{51}\right] \\
+ & \left[0\lambda_{02} + .116\lambda_{12} + .128\lambda_{22} + .136\lambda_{32} + .14\lambda_{42} + .14\lambda_{52}\right] \\
+ & \left[0\lambda_{03} + .18\lambda_{13} + .189\lambda_{23} + .205\lambda_{33} + .223\lambda_{43} + .223\lambda_{53}\right] \\
+ & \left[0\lambda_{04} + .045\lambda_{14} + .045\lambda_{24}\right] \\
+ & \left[0\lambda_{05} + .114\lambda_{15} + .142\lambda_{25} + .150\lambda_{35} + .15\lambda_{45}\right] \\
+ & \left[0\lambda_{06} + .24\lambda_{16} + .252\lambda_{26} + .27\lambda_{36} + .282\lambda_{46} + .293\lambda_{56} + .293\lambda_{66}\right] \\
+ & \left[0\lambda_{07} + .051\lambda_{17} + .053\lambda_{27} + .061\lambda_{37} + .061\lambda_{47}\right] \Big\}
\end{aligned}
$$

CONSTRAINTS

subject to:

$$0\lambda_{01} + .012\lambda_{11} + .021\lambda_{21} + .025\lambda_{31} + .029\lambda_{41} + 1.0\lambda_{51} - X_1 = 0$$

$$0\lambda_{02} + .029\lambda_{12} + .033\lambda_{22} + .037\lambda_{32} + .041\lambda_{42} + 1.0\lambda_{52} - (X_1 + X_2) = 0$$

$$0\lambda_{03} + .045\lambda_{13} + .048\lambda_{23} + .056\lambda_{33} + .074\lambda_{43} + 1.0\lambda_{53} - (X_2 + X_3) = 0$$

$$0\lambda_{04} + .045\lambda_{14} + 1.0\lambda_{24} - X_3 = 0$$

$$0\lambda_{05} + .038\lambda_{15} + .052\lambda_{25} + .060\lambda_{35} + 1.0\lambda_{45} - (X_3 + X_4) = 0$$

$$0\lambda_{06} + .048\lambda_{16} + .051\lambda_{26} + .057\lambda_{36} + .063\lambda_{46} + .074\lambda_{56} + 1.0\lambda_{66} - (X_4 + X_5) = 0$$

$$0\lambda_{07} + .017\lambda_{17} + .018\lambda_{27} + .026\lambda_{37} + 1.0\lambda_{47} - X_5 = 0$$

$$X_1 + X_2 + X_3 + X_4 + X_5 = .125 \qquad \forall X_{i1} \geq 0$$

$$\sum_{i=0}^{5} \lambda_{i1} = 1 \qquad \forall \lambda_{i1} \geq 0$$

$$\sum_{i=0}^{5} \lambda_{i2} = 1 \qquad \forall \lambda_{i2} \geq 0$$

$$\sum_{i=0}^{5} \lambda_{i3} = 1 \qquad \forall \lambda_{i3} \geq 0$$

$$\sum_{i=0}^{2} \lambda_{i4} = 1 \qquad \forall \lambda_{i4} \geq 0$$

$$\sum_{i=0}^{4} \lambda_{i5} = 1 \qquad \forall \lambda_{i5} \geq 0$$

$$\sum_{i=0}^{6} \lambda_{i6} = 1 \qquad \forall \lambda_{i6} \geq 0$$

$$\sum_{i=0}^{4} \lambda_{i7} = 1 \qquad \forall \lambda_{i7} \geq 0$$

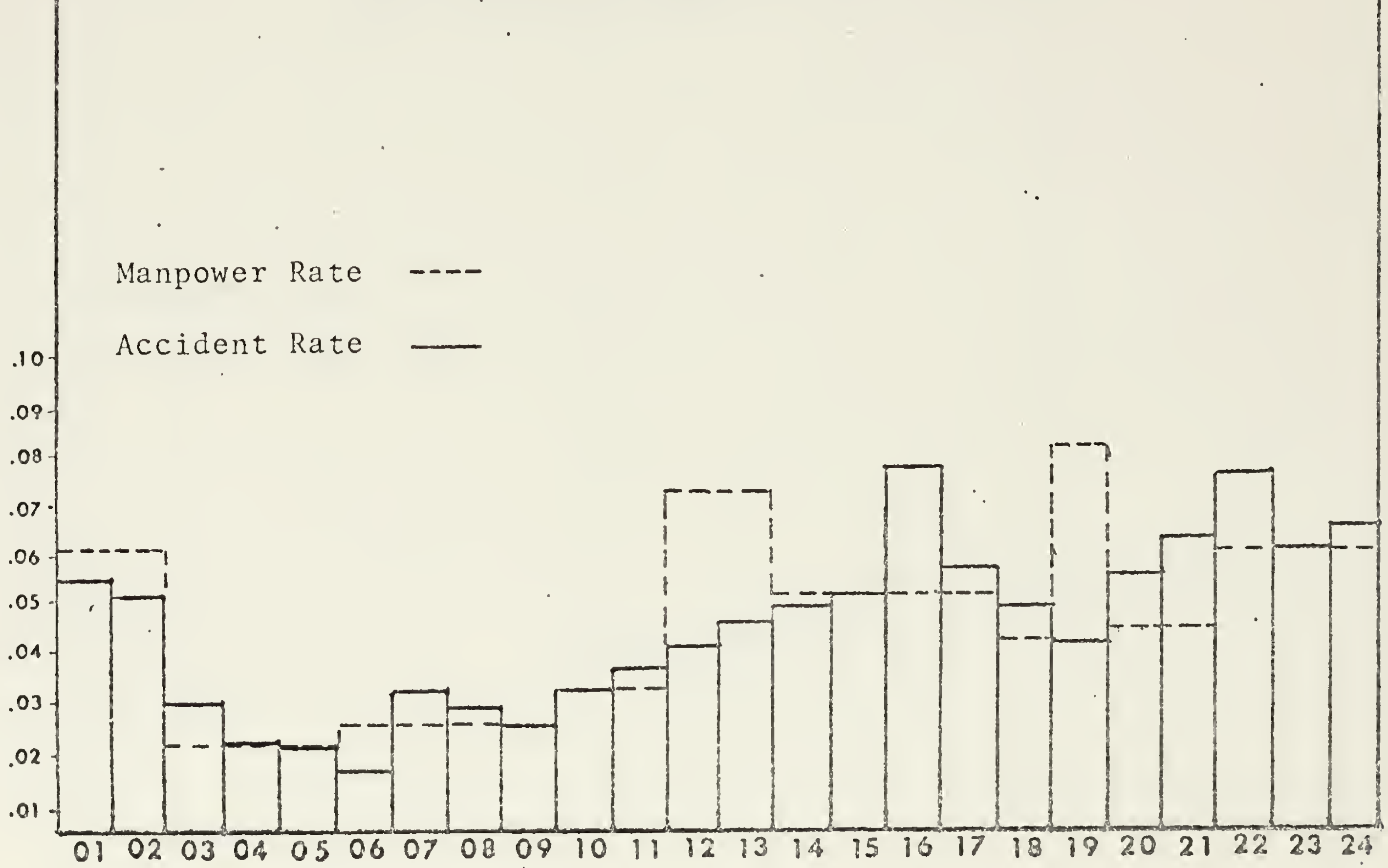
APPENDIX C

MODEL SOLUTION

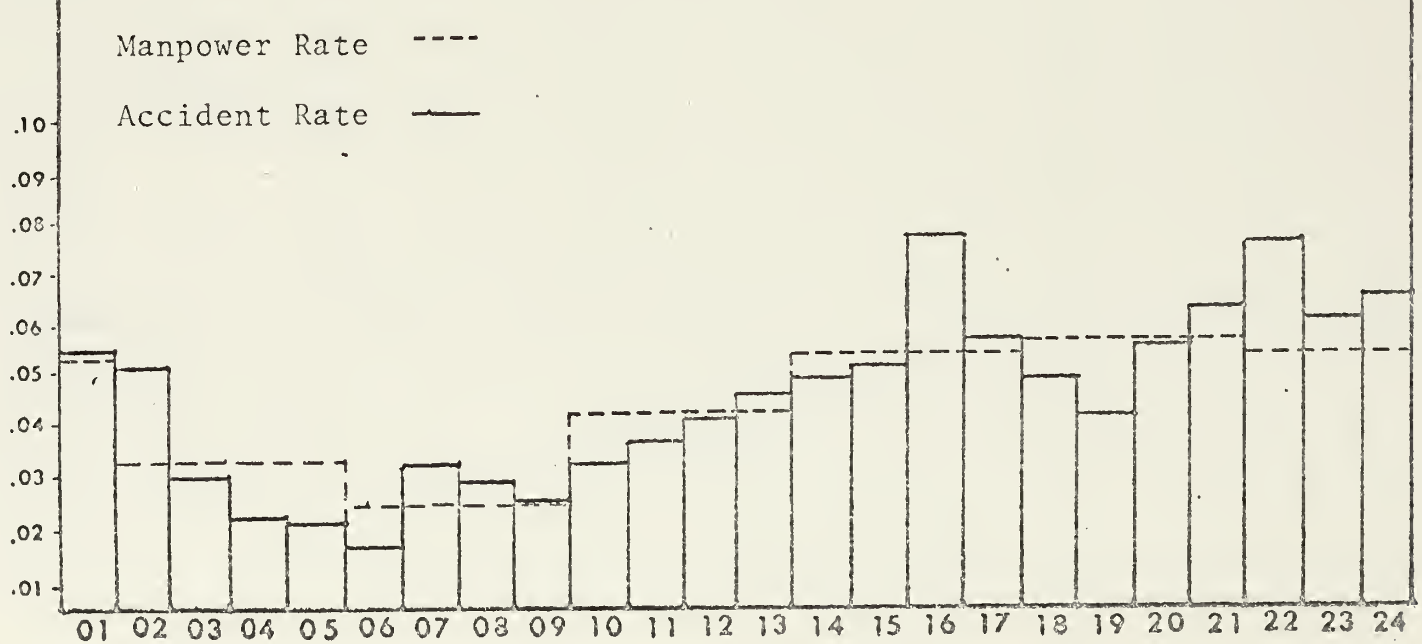
MANPOWER ALLOCATION VS. 1971 ACCIDENT RATE

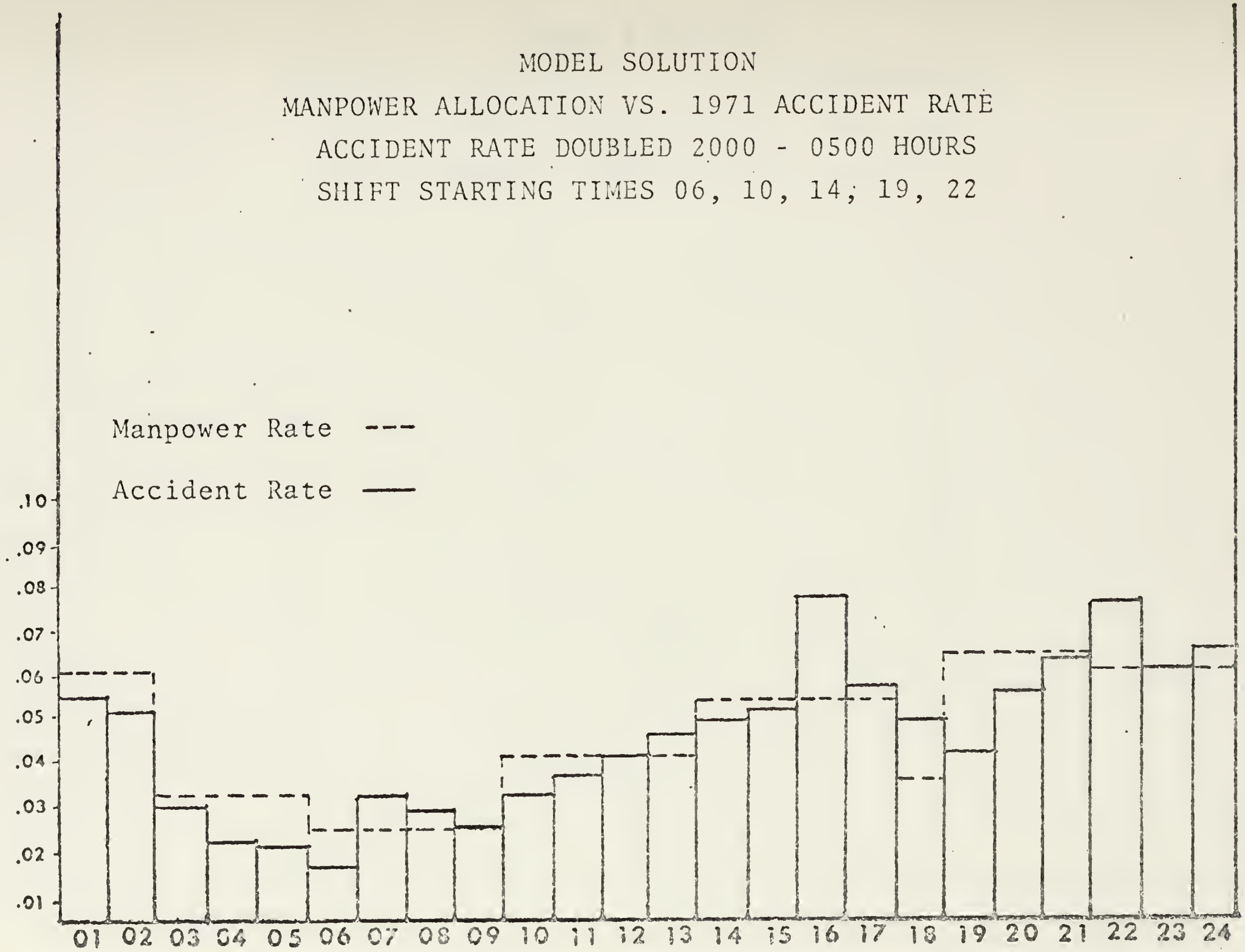
ACCIDENT RATE DOUBLED 2000 - 0500 HOURS

SHIFT STARTING TIMES 06, 10, 14, 19, 22



MODEL SOLUTION
MANPOWER ALLOCATION VS. 1971 ACCIDENT RATE
ACCIDENT RATE DOUBLED 2000 - 0500 HOURS
SHIFT STARTING TIMES 06, 10, 14, 18, 22




MODEL SOLUTION
MANPOWER ALLOCATION VS. 1971 ACCIDENT RATE
ACCIDENT RATE DOUBLED 2000 - 0500 HOURS
SHIFT STARTING TIMES 06, 10, 14, 19, 22
Manpower Rate ---
Accident Rate ——
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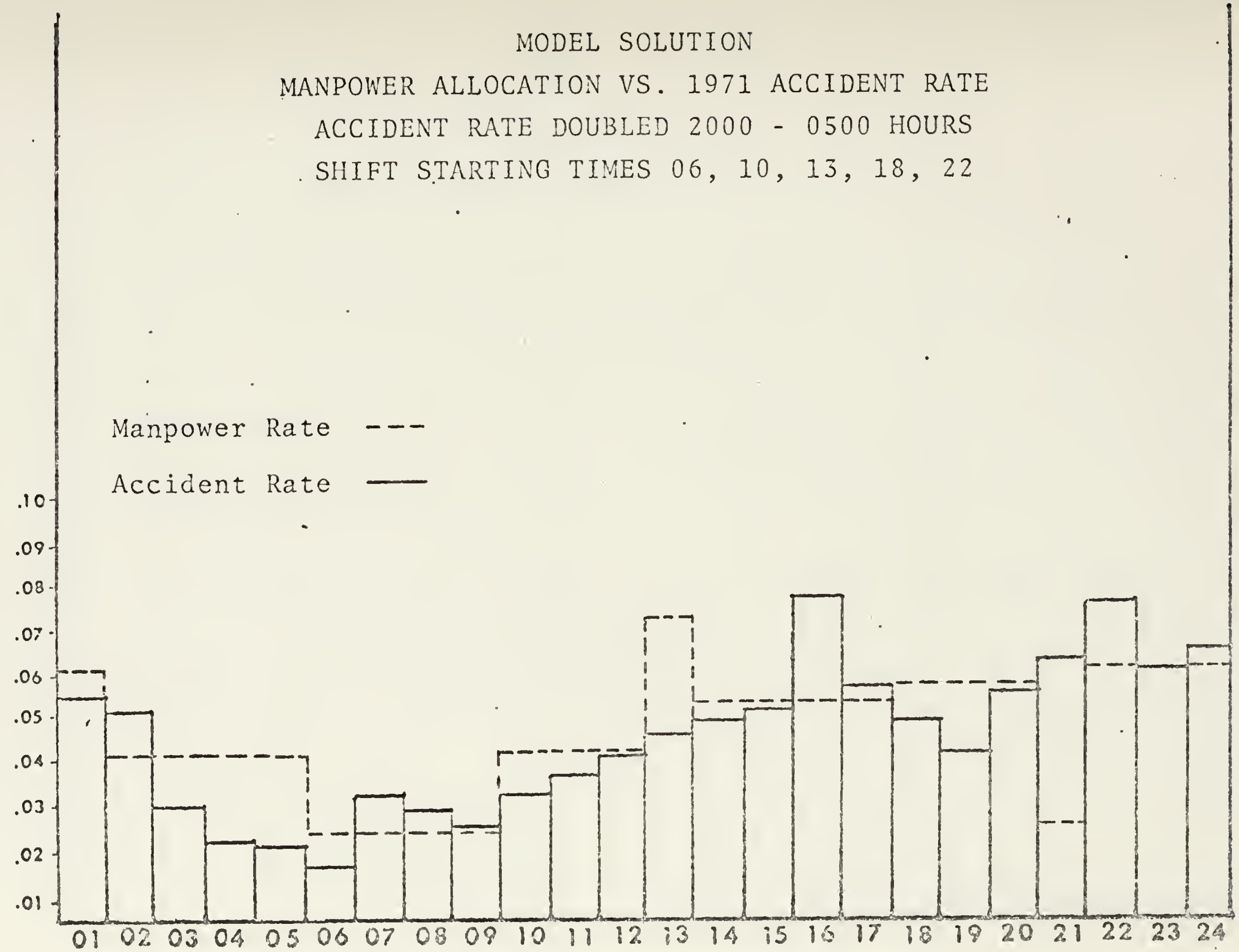
MODEL SOLUTION
MANPOWER ALLOCATION VS. 1971 ACCIDENT RATE
ACCIDENT RATE DOUBLED 2000 - 0500 HOURS
SHIFT STARTING TIMES 06, 10, 13, 18, 22
Manpower Rate
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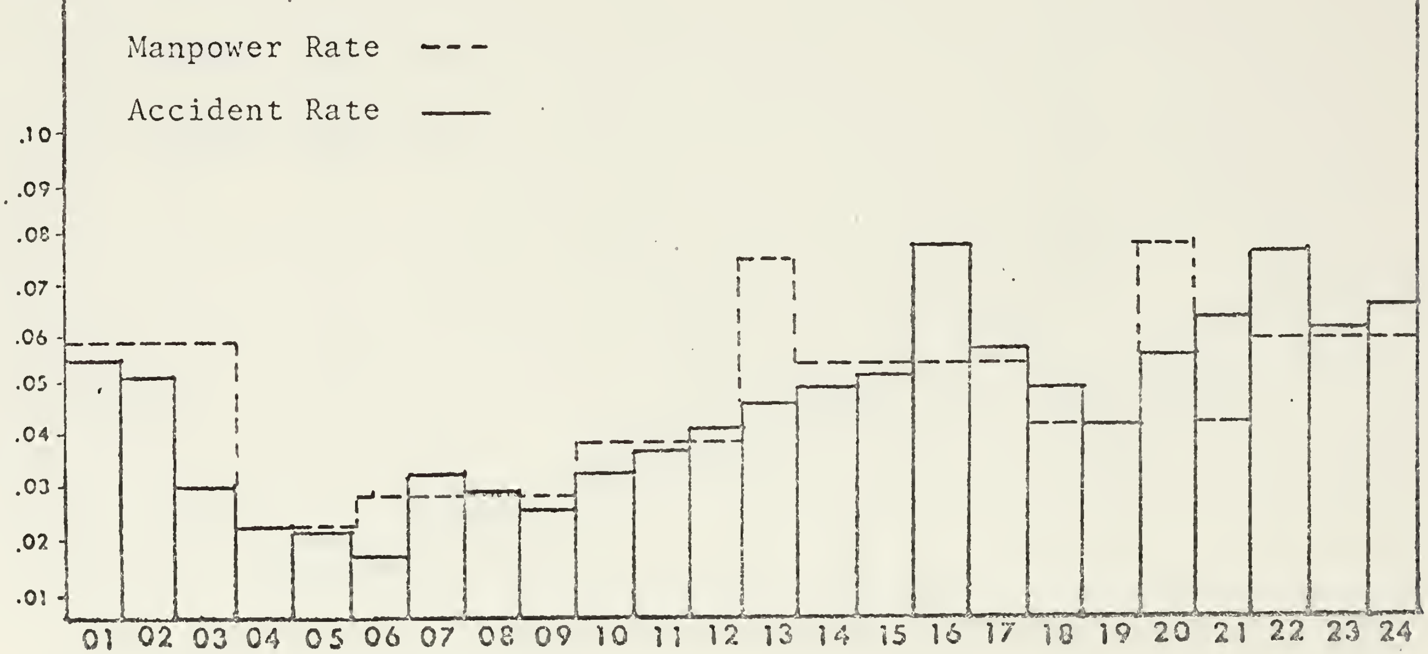
MODEL SOLUTION

MANPOWER ALLOCATION VS. 1971 ACCIDENT RATE

ACCIDENT RATE DOUBLED 2000 - 0500 HOURS

SHIFT STARTING TIMES 06, 10, 13, 20, 22

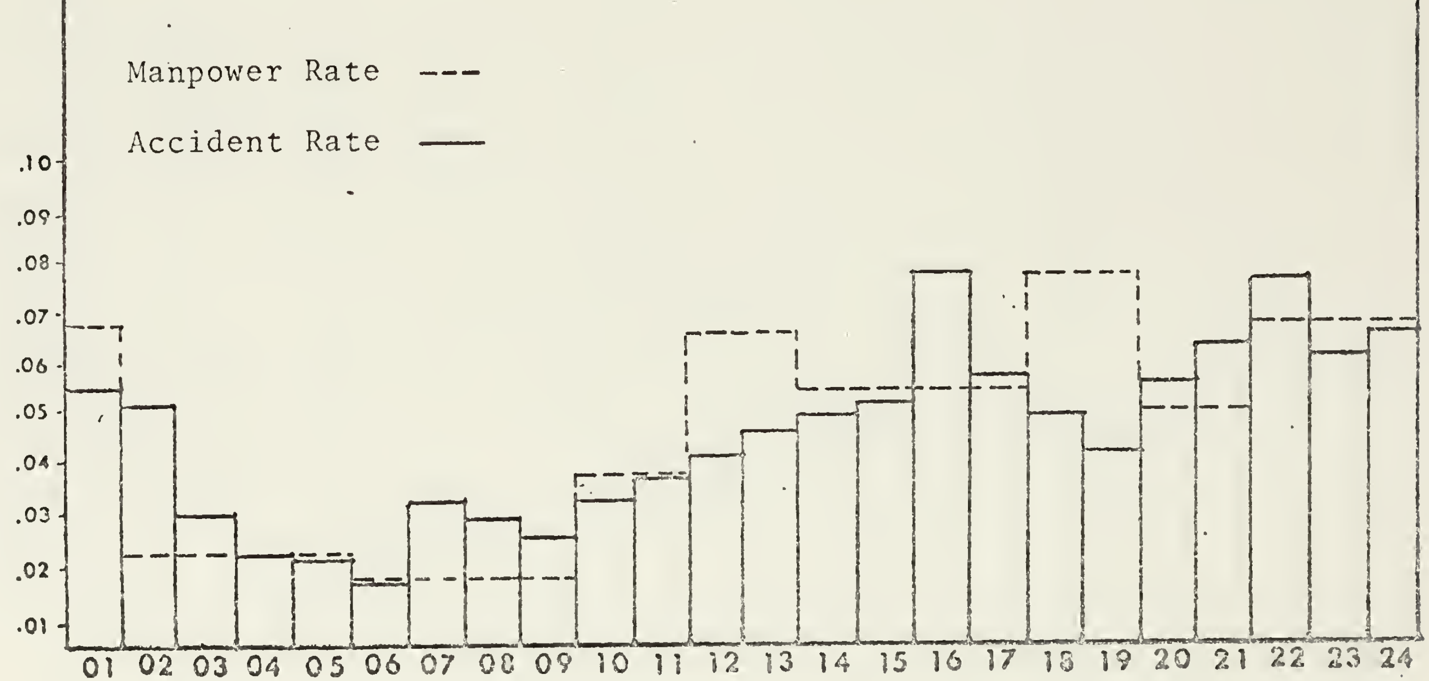


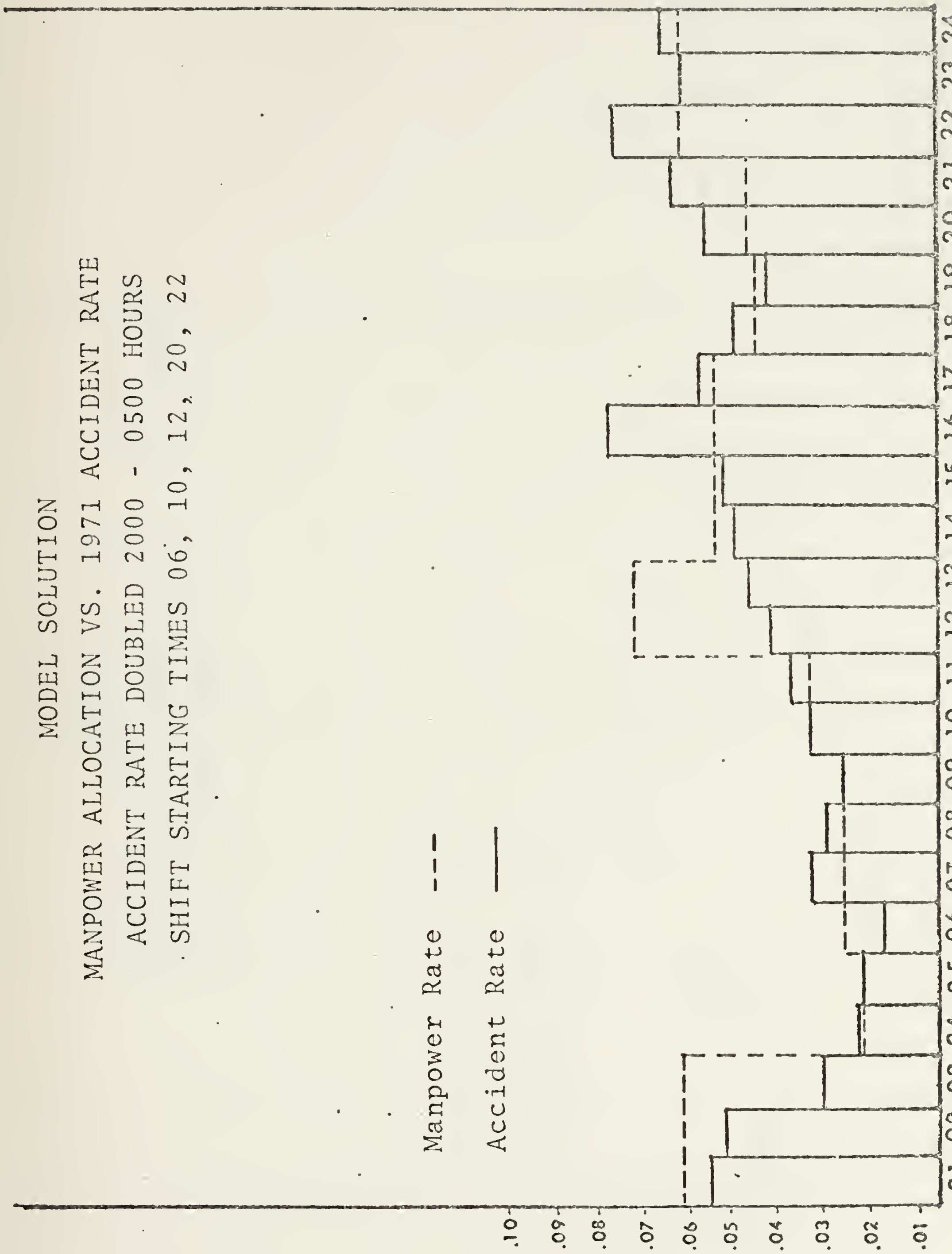
MODEL SOLUTION

MANPOWER ALLOCATION VS. 1971 ACCIDENT RATE

ACCIDENT RATE DOUBLED 2000 - 0500 HOURS

SHIFT STARTING TIMES 06, 10, 12, 18, 22





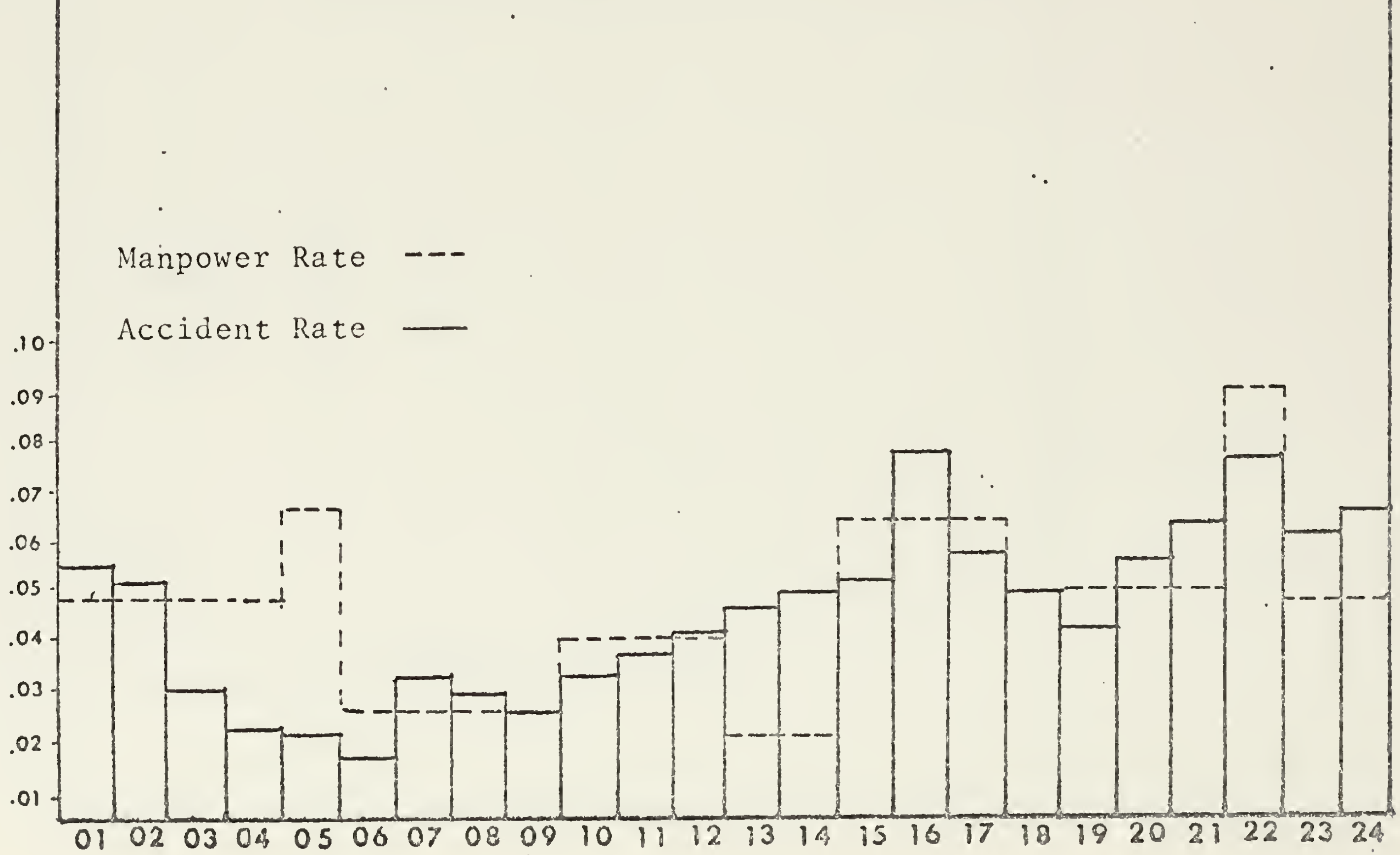
MODEL SOLUTION
MANPOWER ALLOCATION VS. 1971 ACCIDENT RATE
ACCIDENT RATE DOUBLED 2000 - 0500 HOURS
SHIFT STARTING TIMES 06, 10, 12, 20, 22

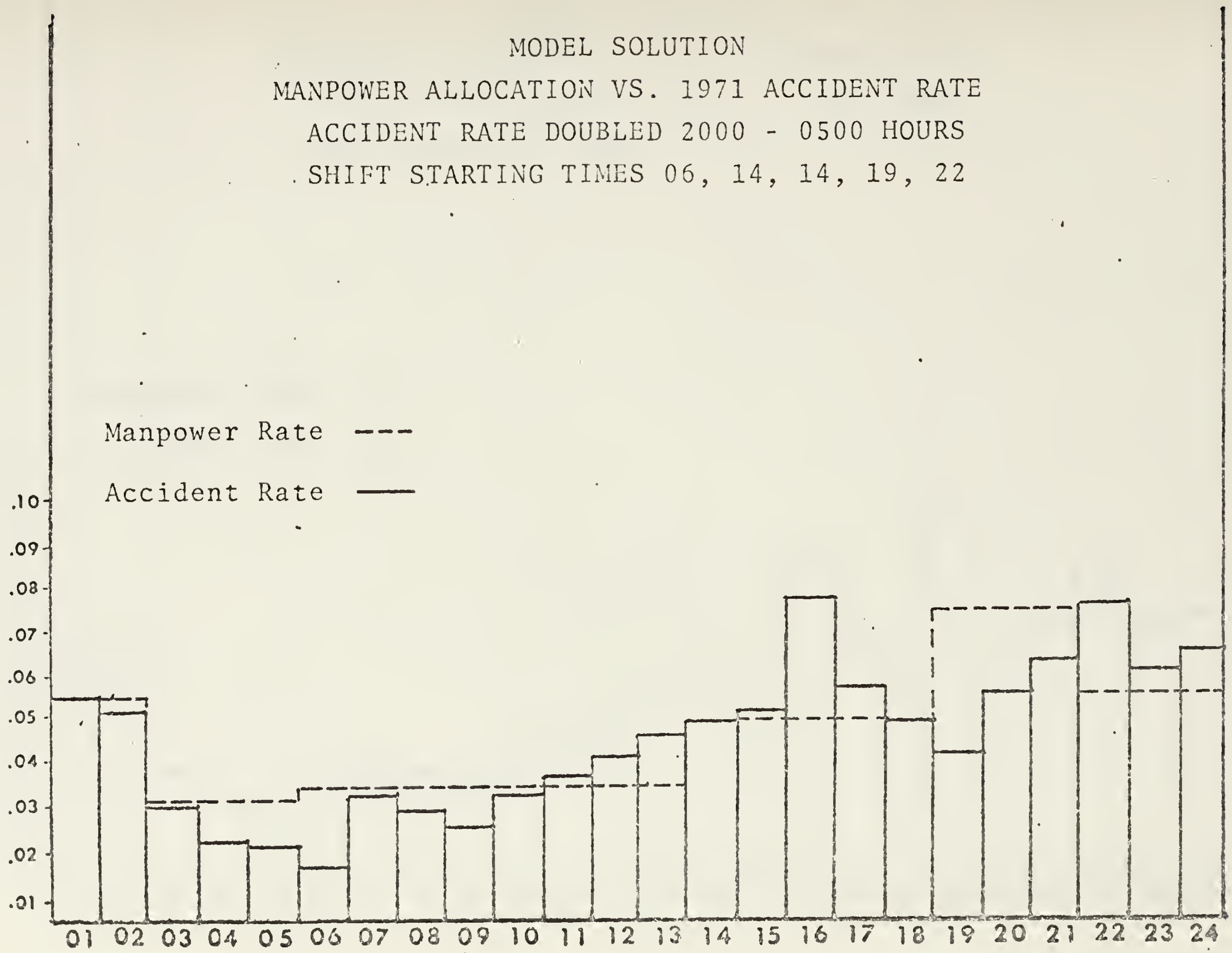
MODEL SOLUTION

MANPOWER ALLOCATION VS. 1971 ACCIDENT RATE

ACCIDENT RATE DOUBLED 2000 - 0500 HOURS

SHIFT STARTING TIMES 05, 10, 15, 22, 22




MODEL SOLUTION
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ACCIDENT RATE DOUBLED 2000 - 0500 HOURS
SHIFT STARTING TIMES 06, 14, 14, 19, 22
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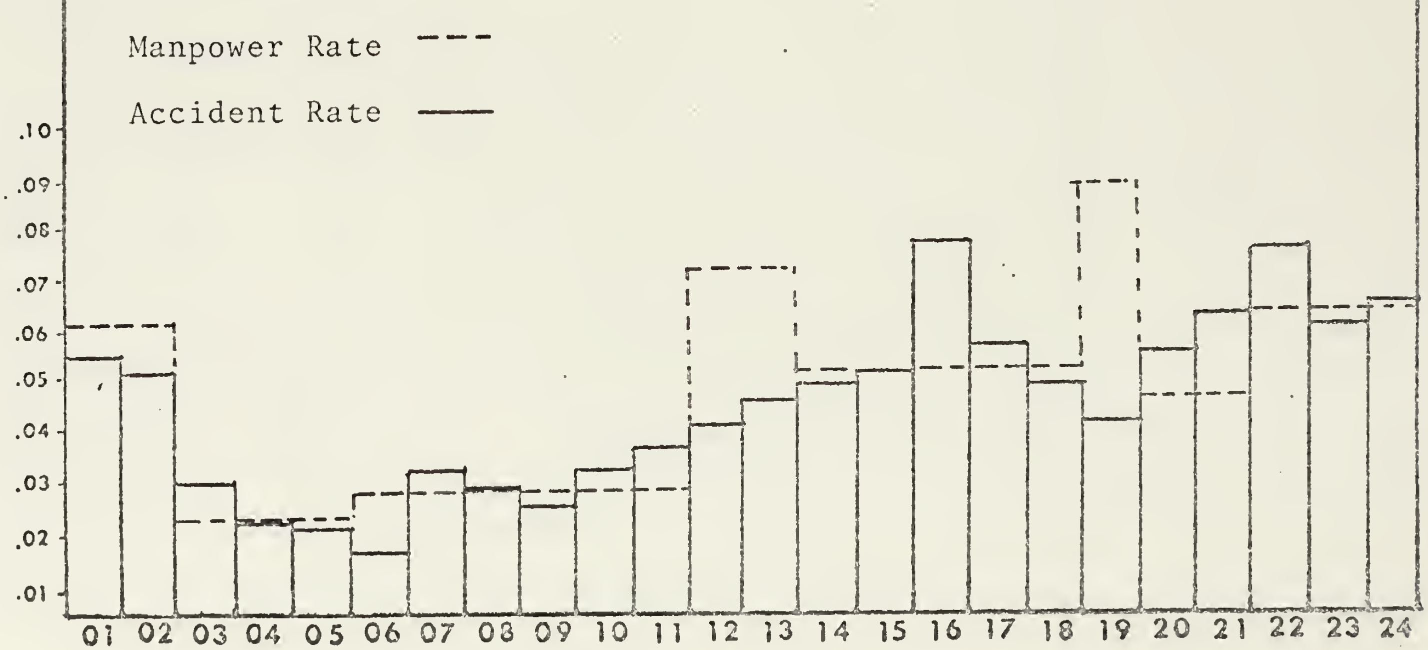
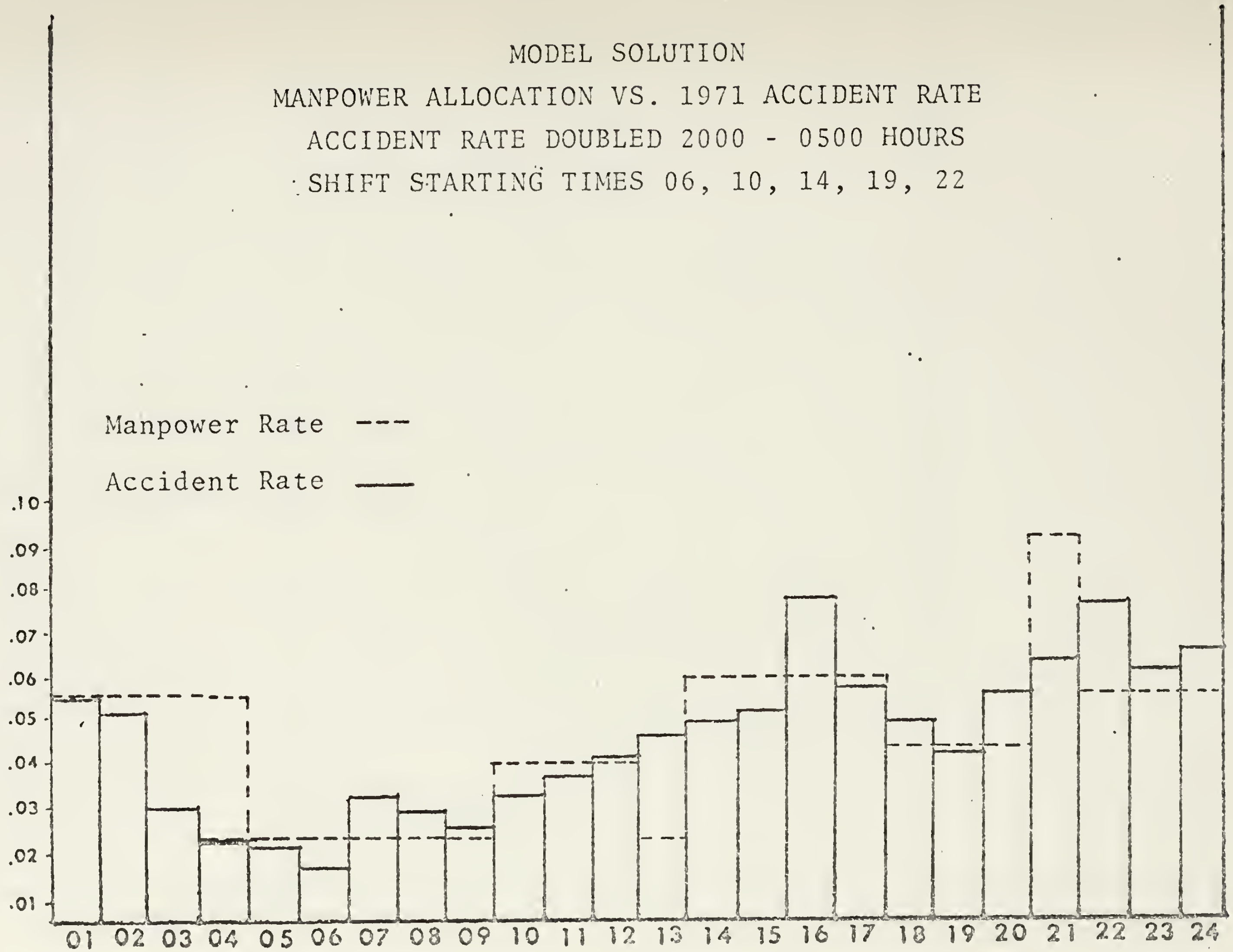
MODEL SOLUTION

MANPOWER ALLOCATION VS. 1971 ACCIDENT RATE

ACCIDENT RATE DOUBLED 2000 - 0500 HOURS

SHIFT STARTING TIMES 06, 12, 12, 19, 22




MODEL SOLUTION
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SHIFT STARTING TIMES 06, 10, 14, 19, 22
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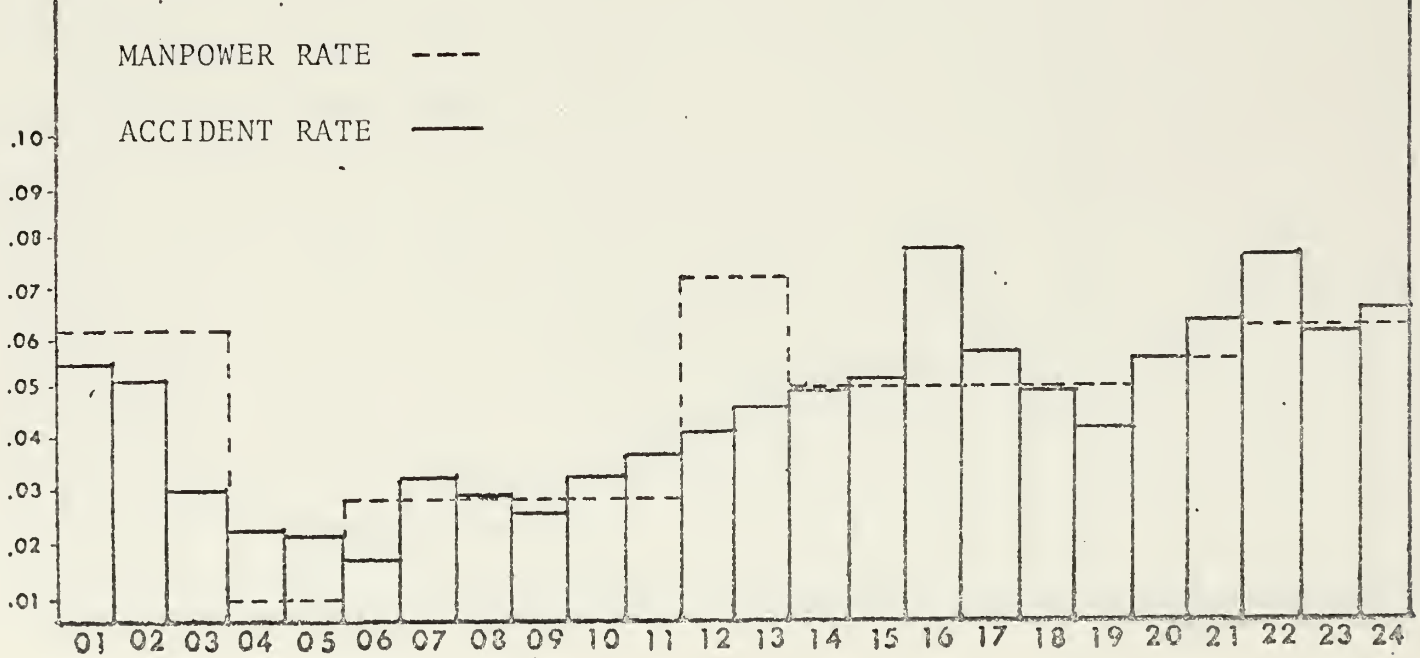
MODEL SOLUTION

MANPOWER ALLOCATION VS. 1971 ACCIDENT RATE

ACCIDENT RATE DOUBLED 2000 - 0500 HOURS

SHIFT STARTING TIMES 06, 12, 12, 20, 22

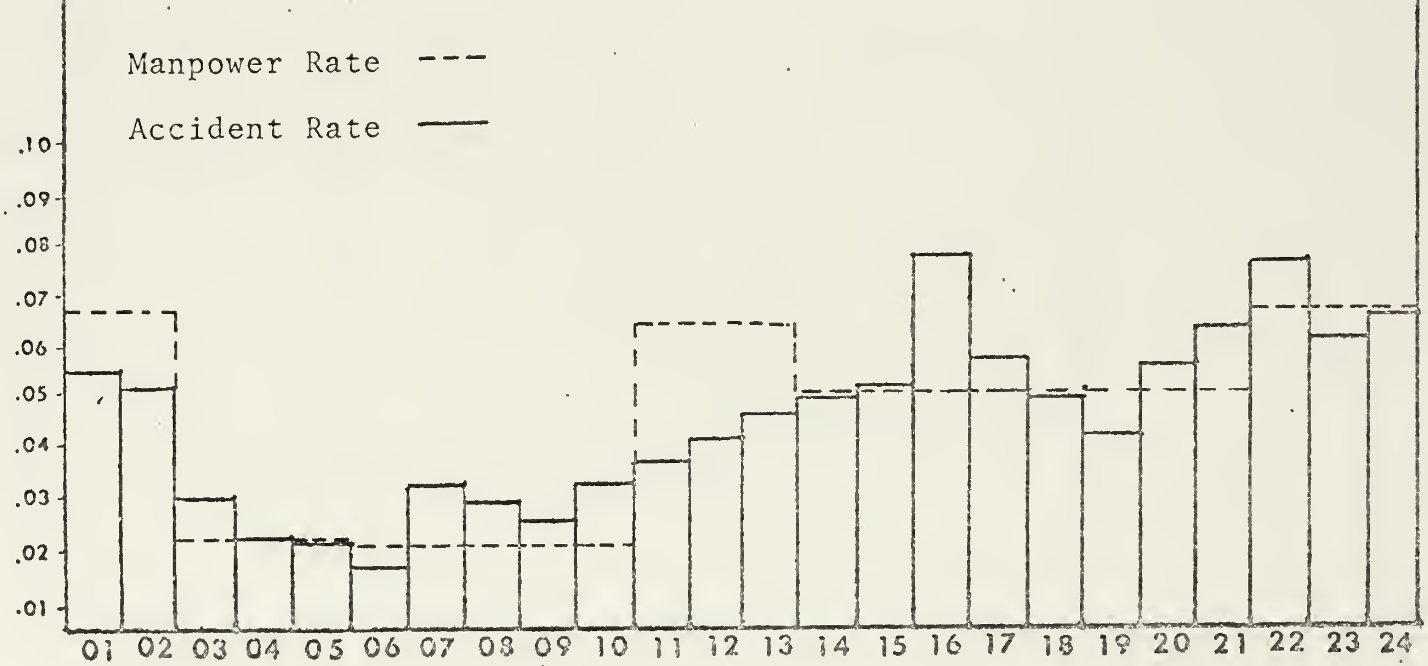


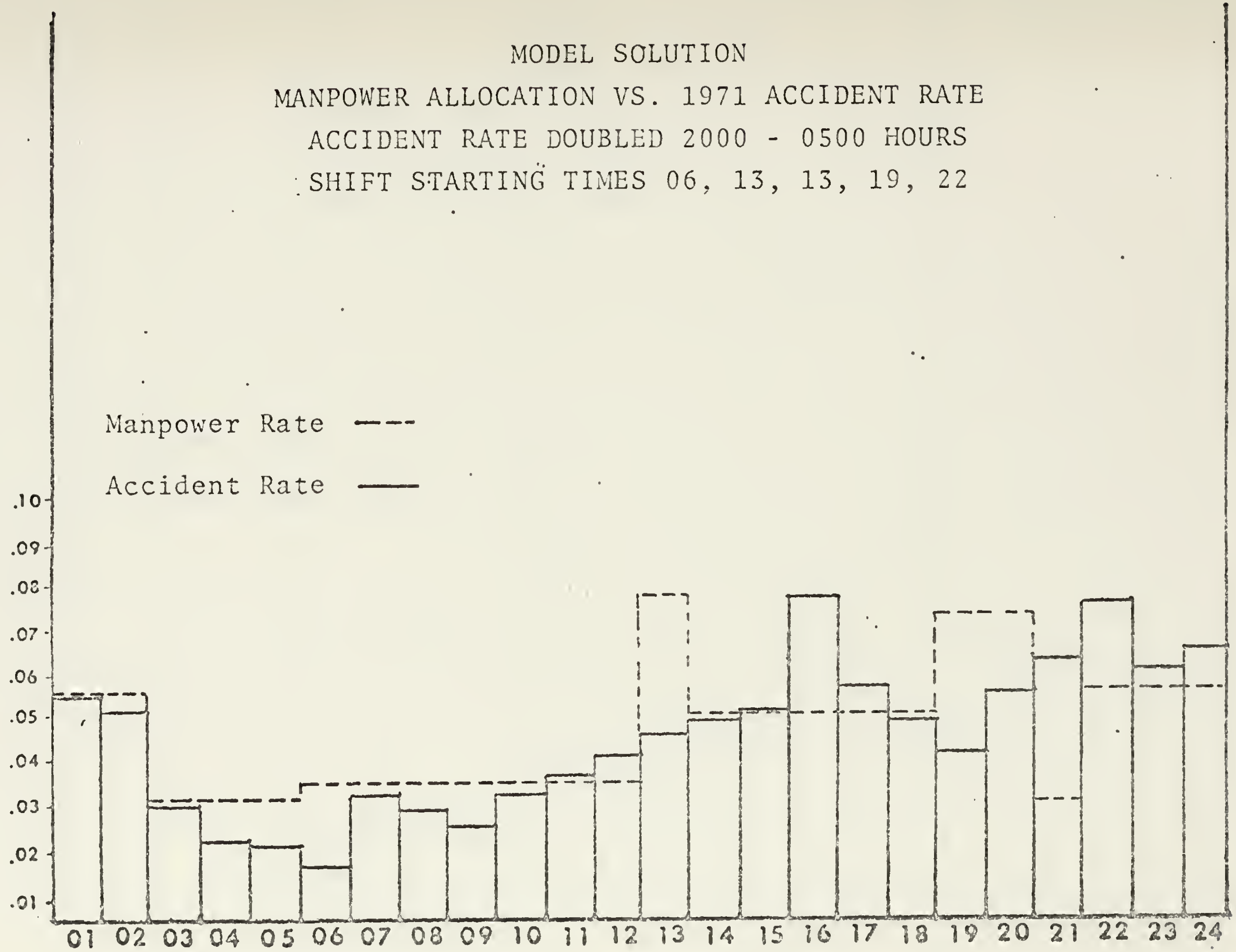
MODEL SOLUTION

MANPOWER ALLOCATION VS. 1971 ACCIDENT RATE

ACCIDENT RATE DOUBLED 2000 - 0500 HOURS

SHIFT STARTING TIMES 06, 11, 11, 19, 22




MODEL SOLUTION
MANPOWER ALLOCATION VS. 1971 ACCIDENT RATE
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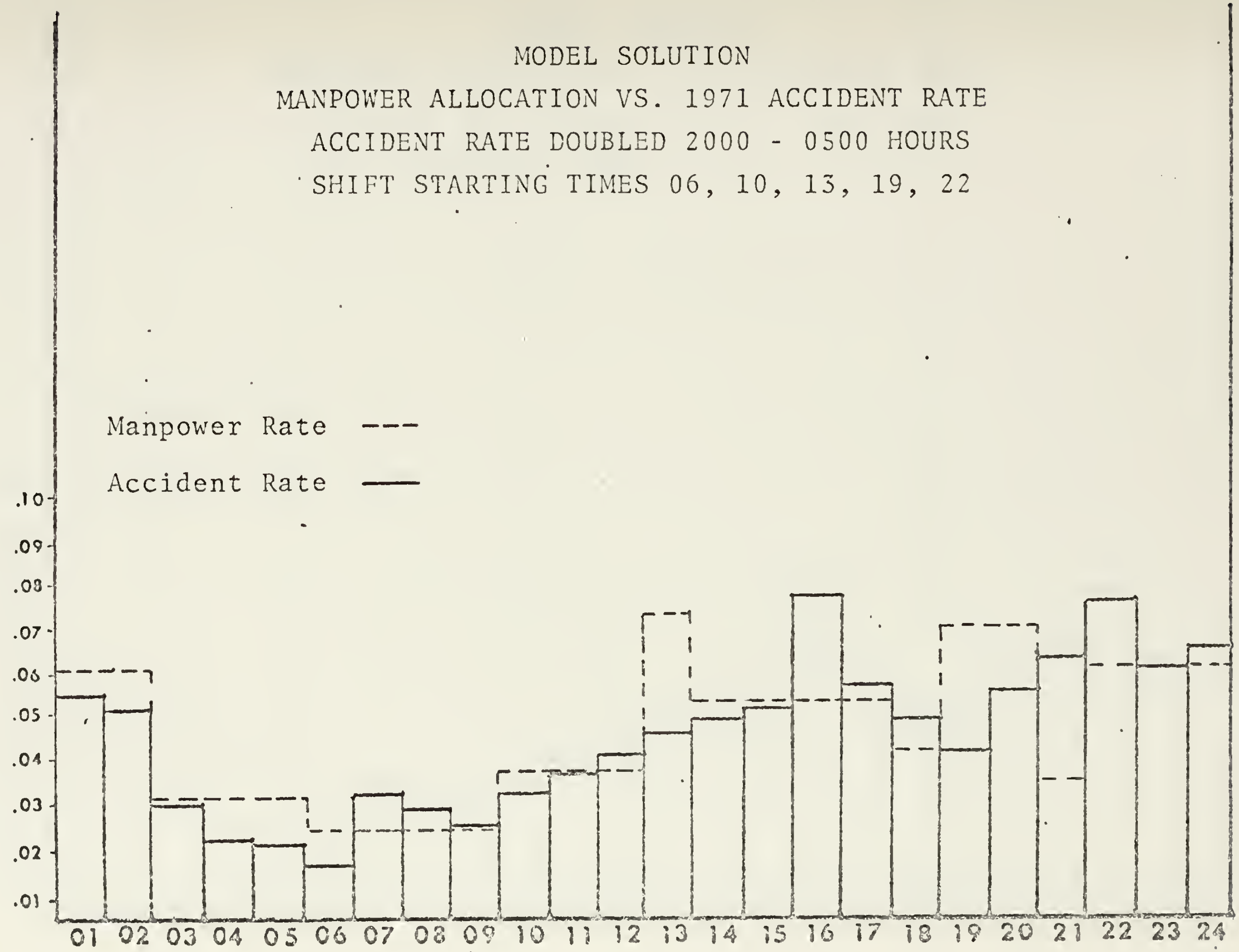
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MODEL SOLUTION

MANPOWER ALLOCATION VS. 1971 ACCIDENT RATE

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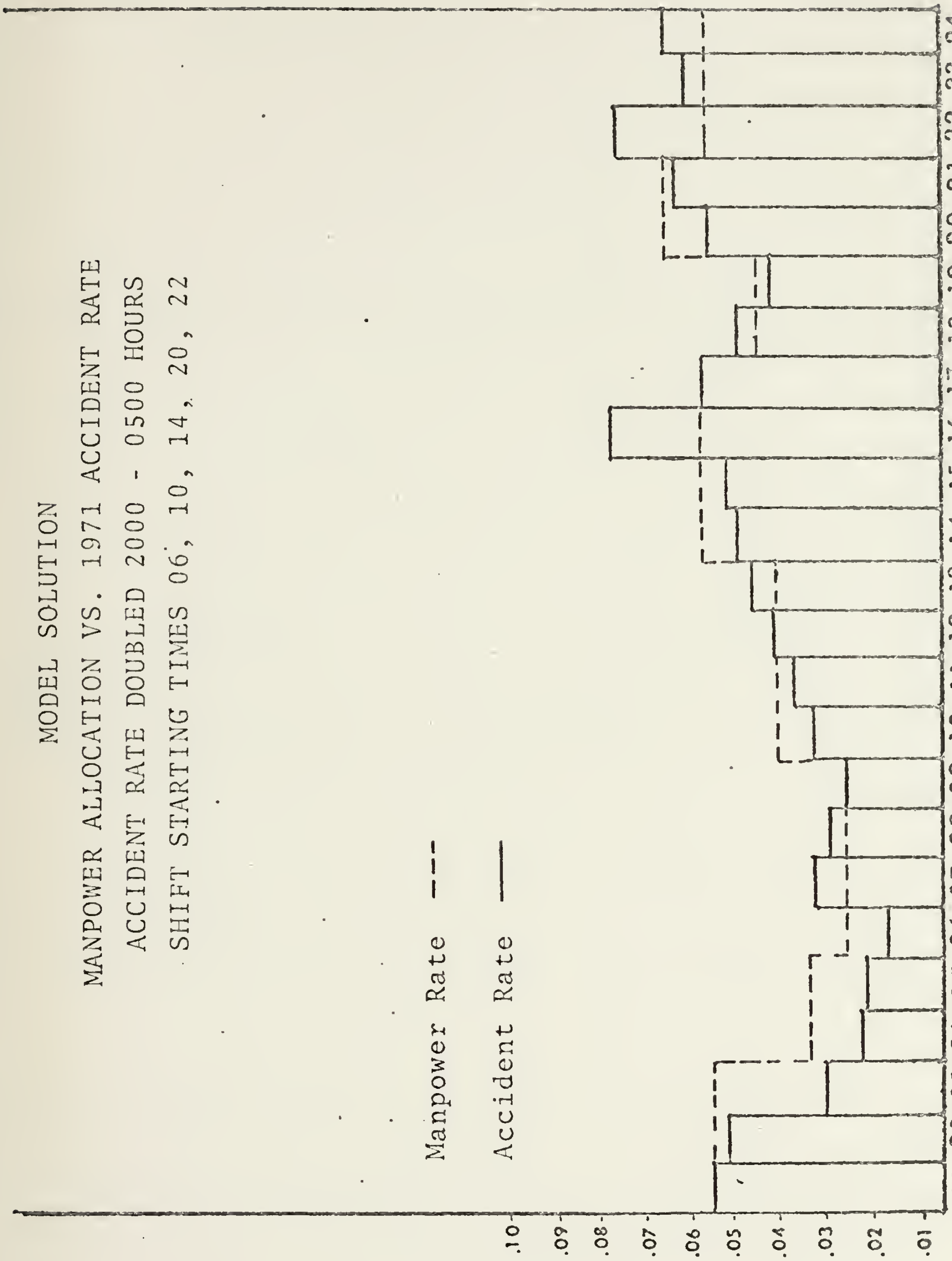
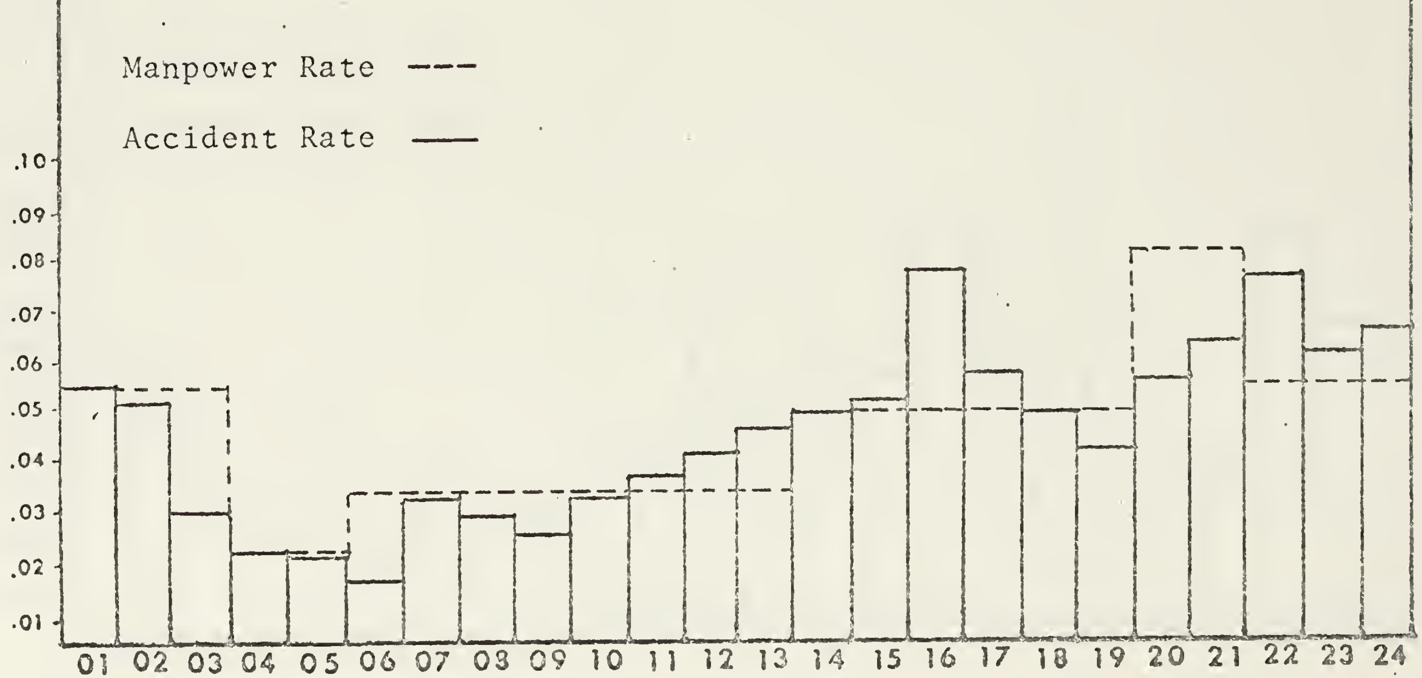


MODEL SOLUTION

MANPOWER ALLOCATION VS. 1971 ACCIDENT RATE

ACCIDENT RATE DOUBLED 2000 - 0500 HOURS

SHIFT STARTING TIMES 06, 14, 14, 20, 22



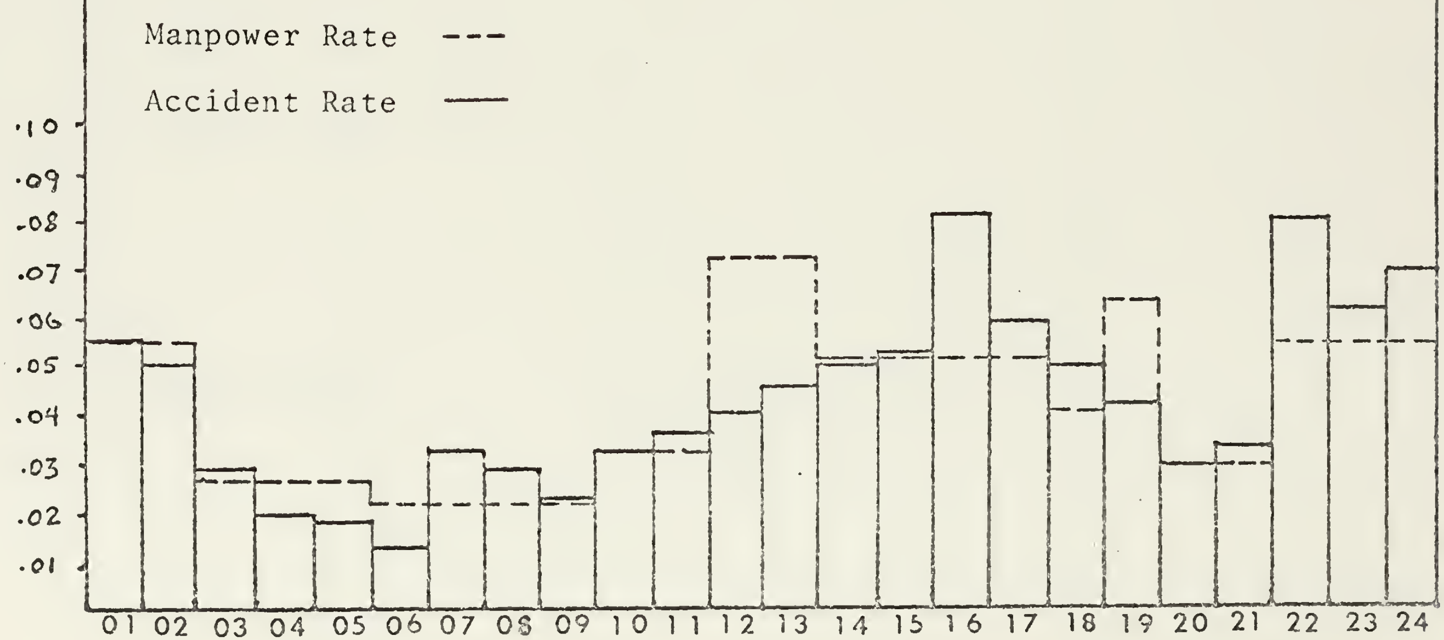
APPENDIX D

MODEL SOLUTION

MANPOWER ALLOCATION VS. 1971 ACCIDENT RATE

ACCIDENT RATE DOUBLED 2200 - 0500 HOURS

SHIFT STARTING TIMES 06, 10, 14, 19, 22

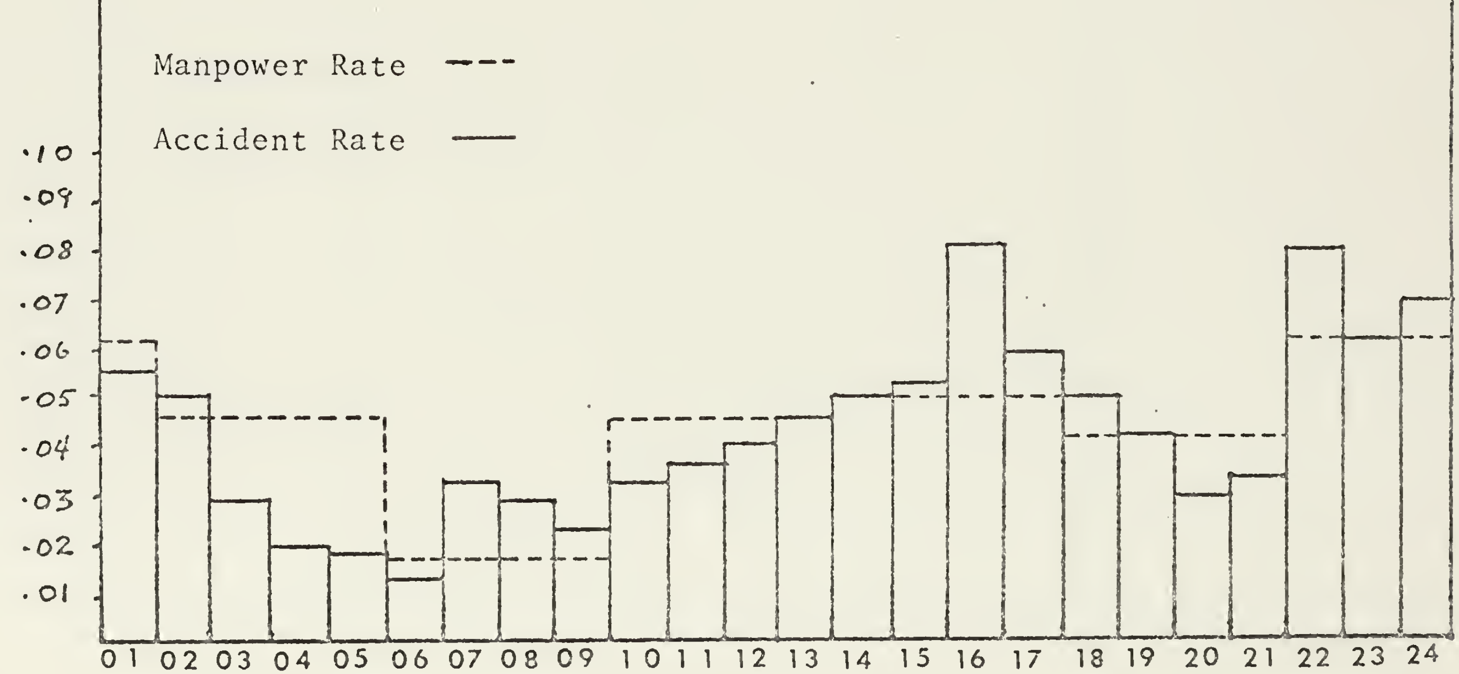


MODEL SOLUTION

MANPOWER ALLOCATION VS. 1971 ACCIDENT RATE

ACCIDENT RATE DOUBLED 2200 - 0500 HOURS

SHIFT STARTING TIMES 06, 10, 14, 18, 22



MODEL SOLUTION

MANPOWER ALLOCATION VS. 1971 ACCIDENT RATE

ACCIDENT RATE DOUBLED 2200 - 0500 HOURS

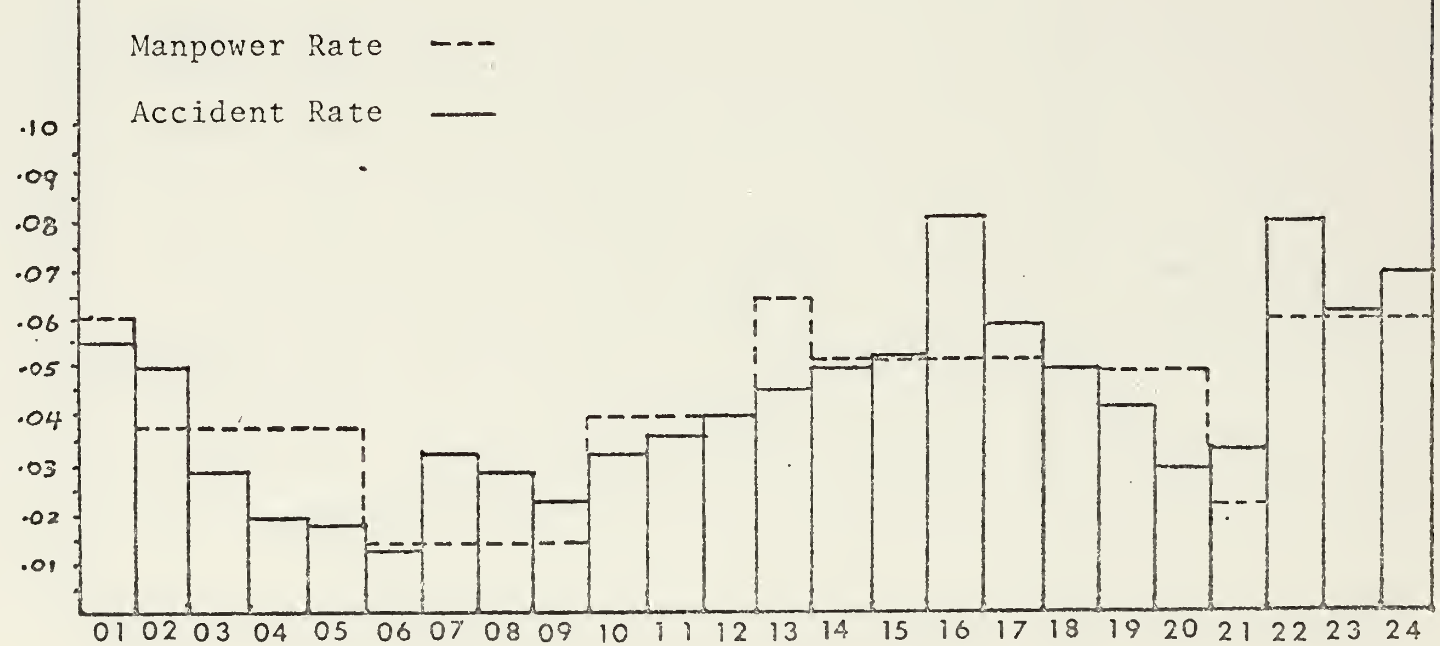
SHIFT STARTING TIMES 06, 10, 14, 19, 22

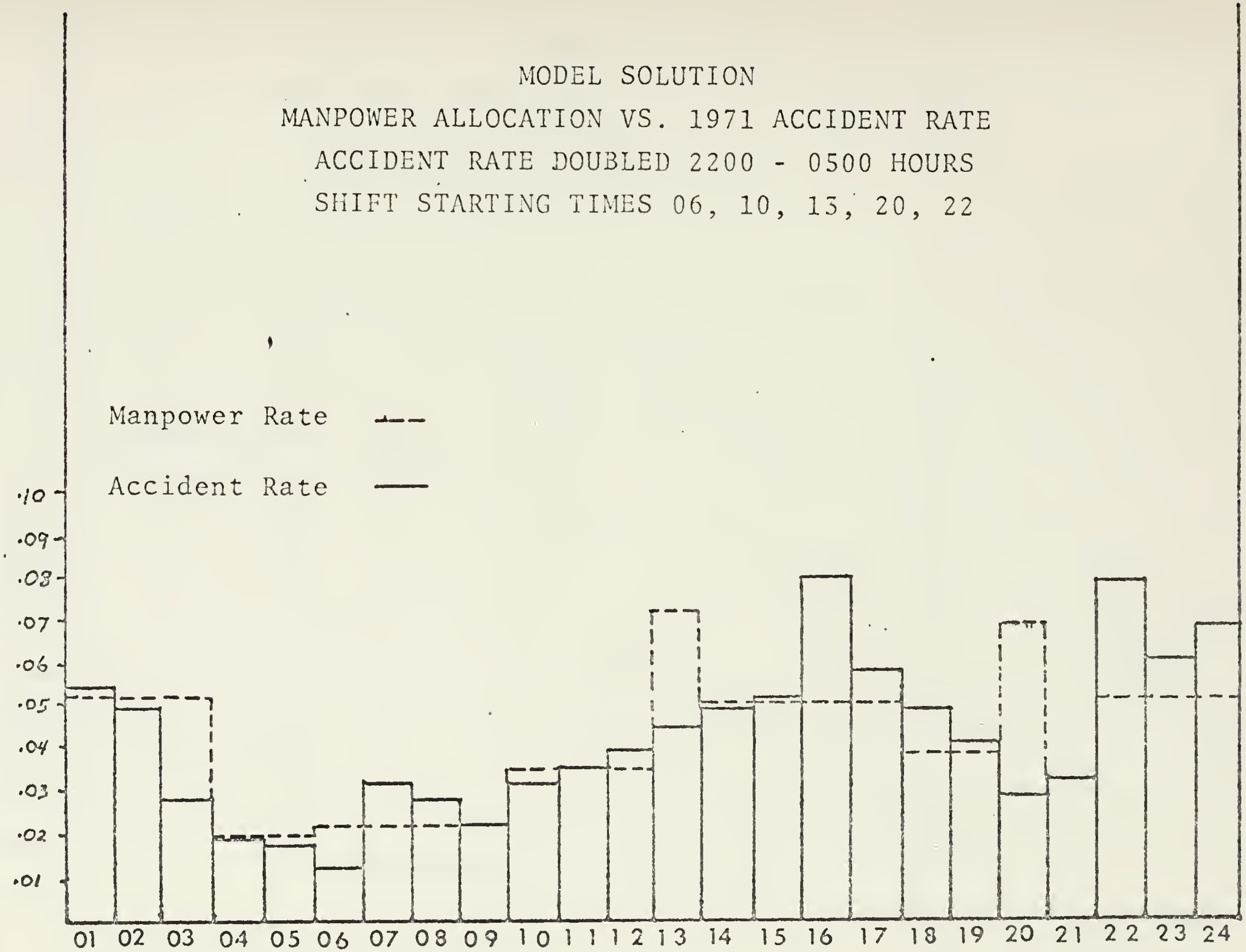
MODEL SOLUTION

MANPOWER ALLOCATION VS. 1971 ACCIDENT RATE

ACCIDENT RATE DOUBLED 2200 - 0500 HOURS

SHIFT STARTING TIMES 06, 10, 13, 18, 22




MODEL SOLUTION
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ACCIDENT RATE DOUBLED 2200 - 0500 HOURS
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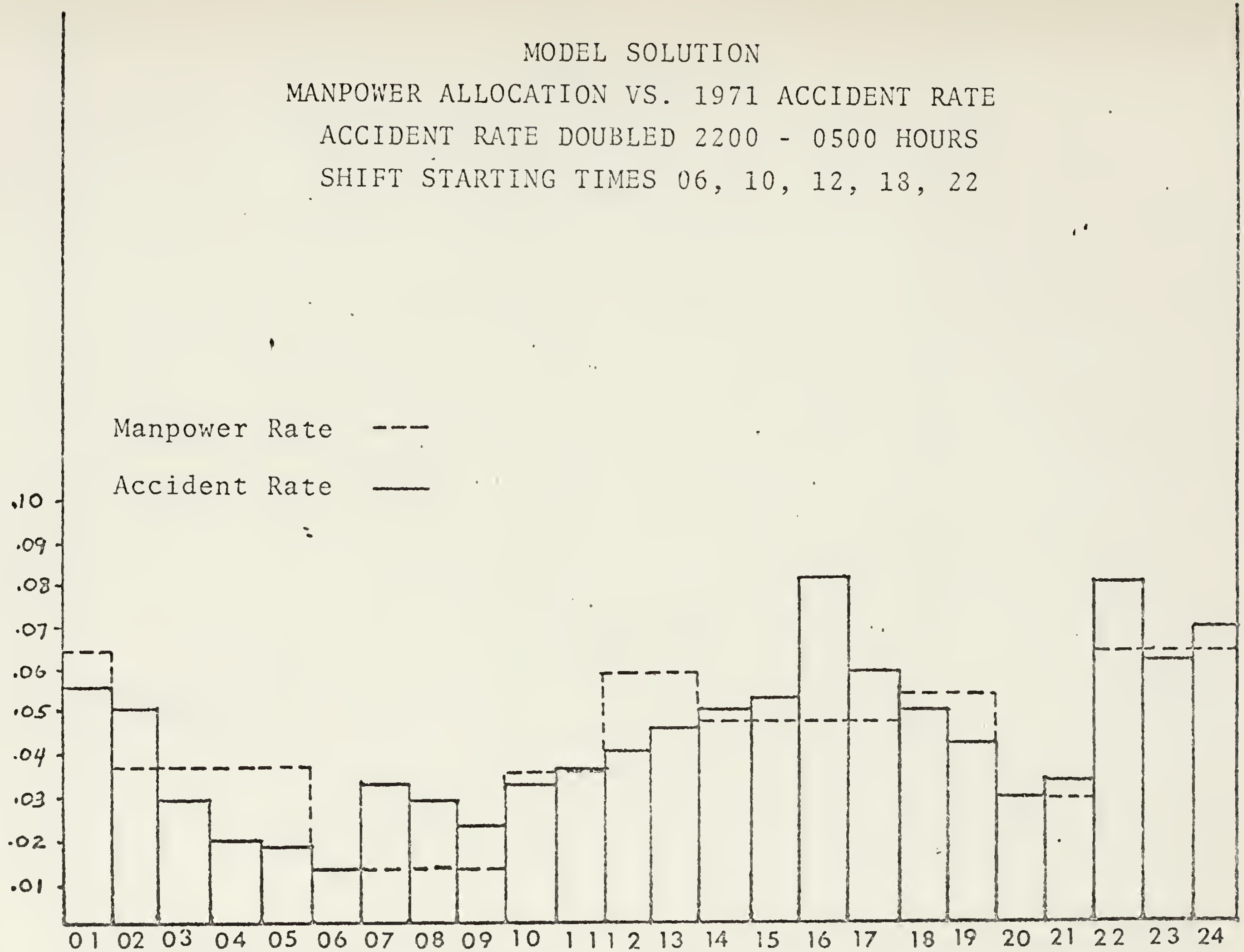
MODEL SOLUTION

MANPOWER ALLOCATION VS. 1971 ACCIDENT RATE

ACCIDENT RATE DOUBLED 2200 - 0500 HOURS

SHIFT STARTING TIMES 06, 10, 12, 18, 22

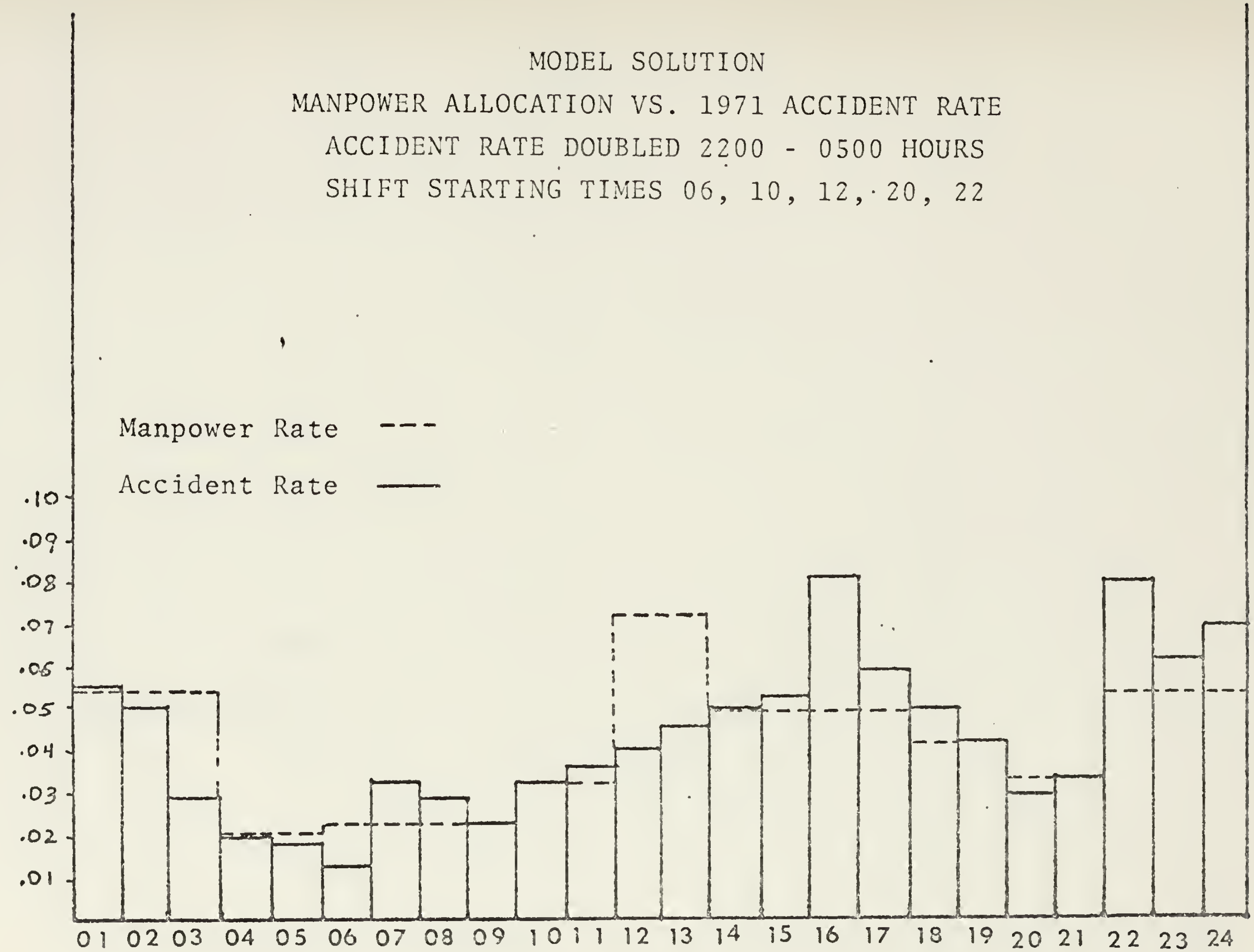


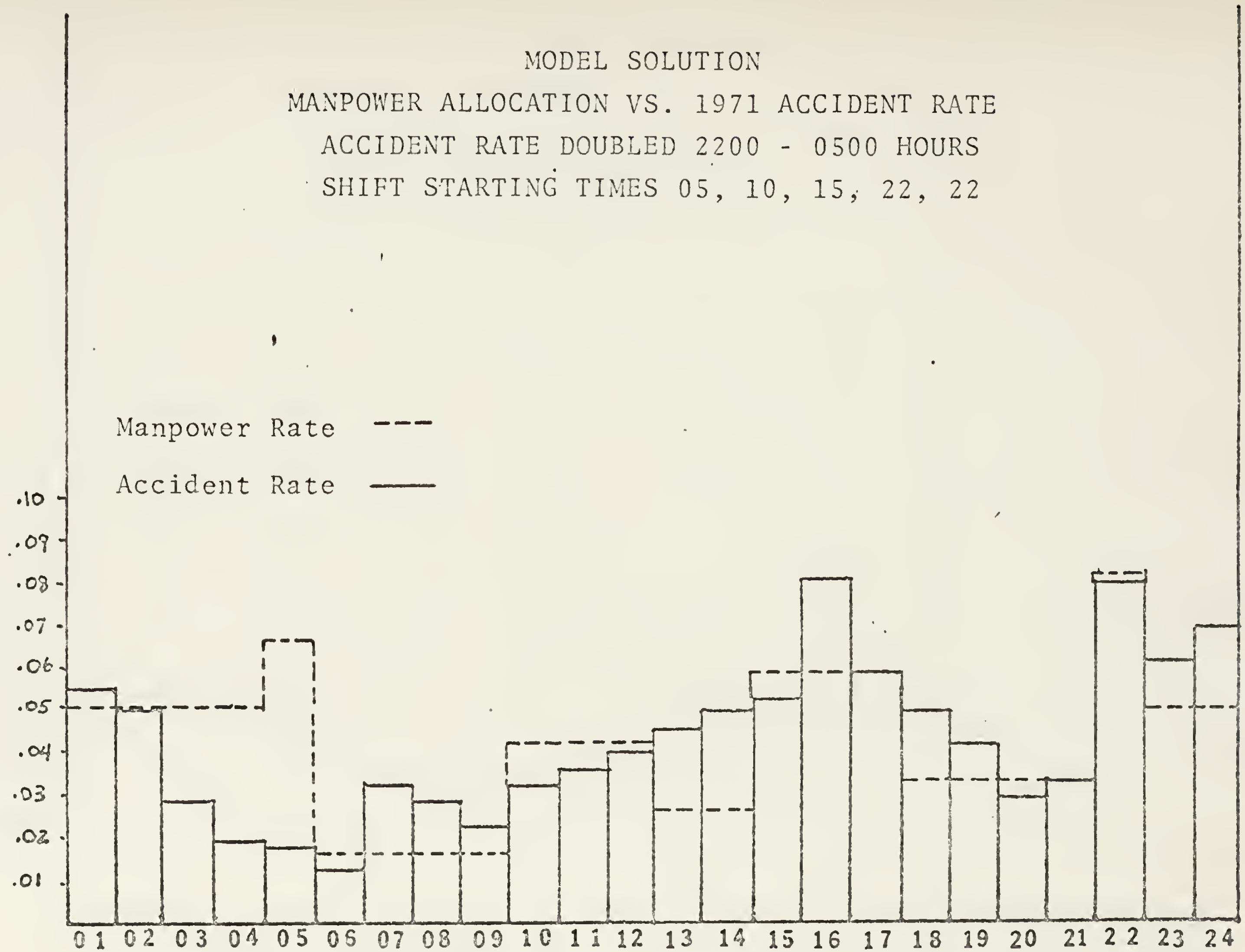
MODEL SOLUTION

MANPOWER ALLOCATION VS. 1971 ACCIDENT RATE

ACCIDENT RATE DOUBLED 2200 - 0500 HOURS

SHIFT STARTING TIMES 06, 10, 12, 20, 22

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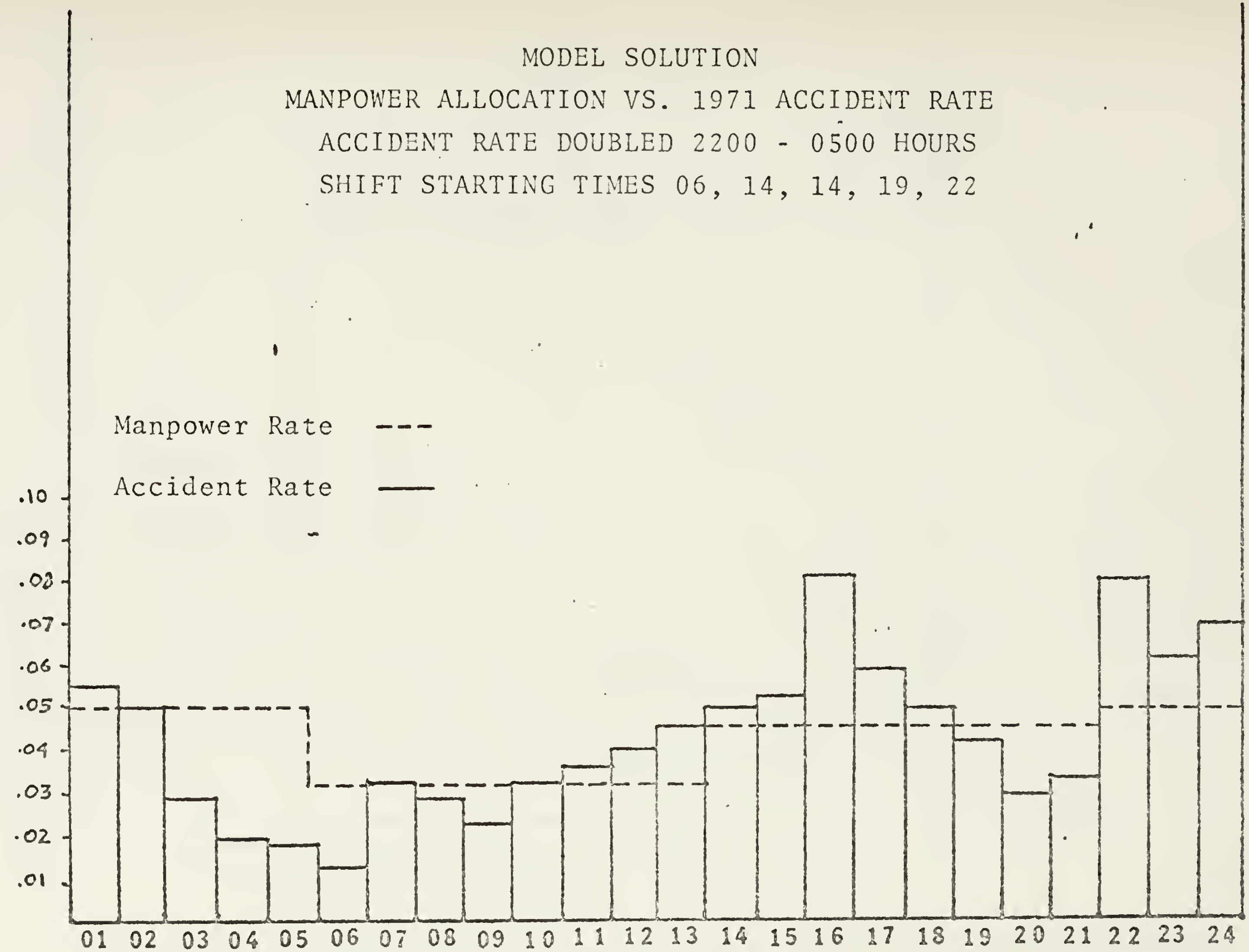
MODEL SOLUTION
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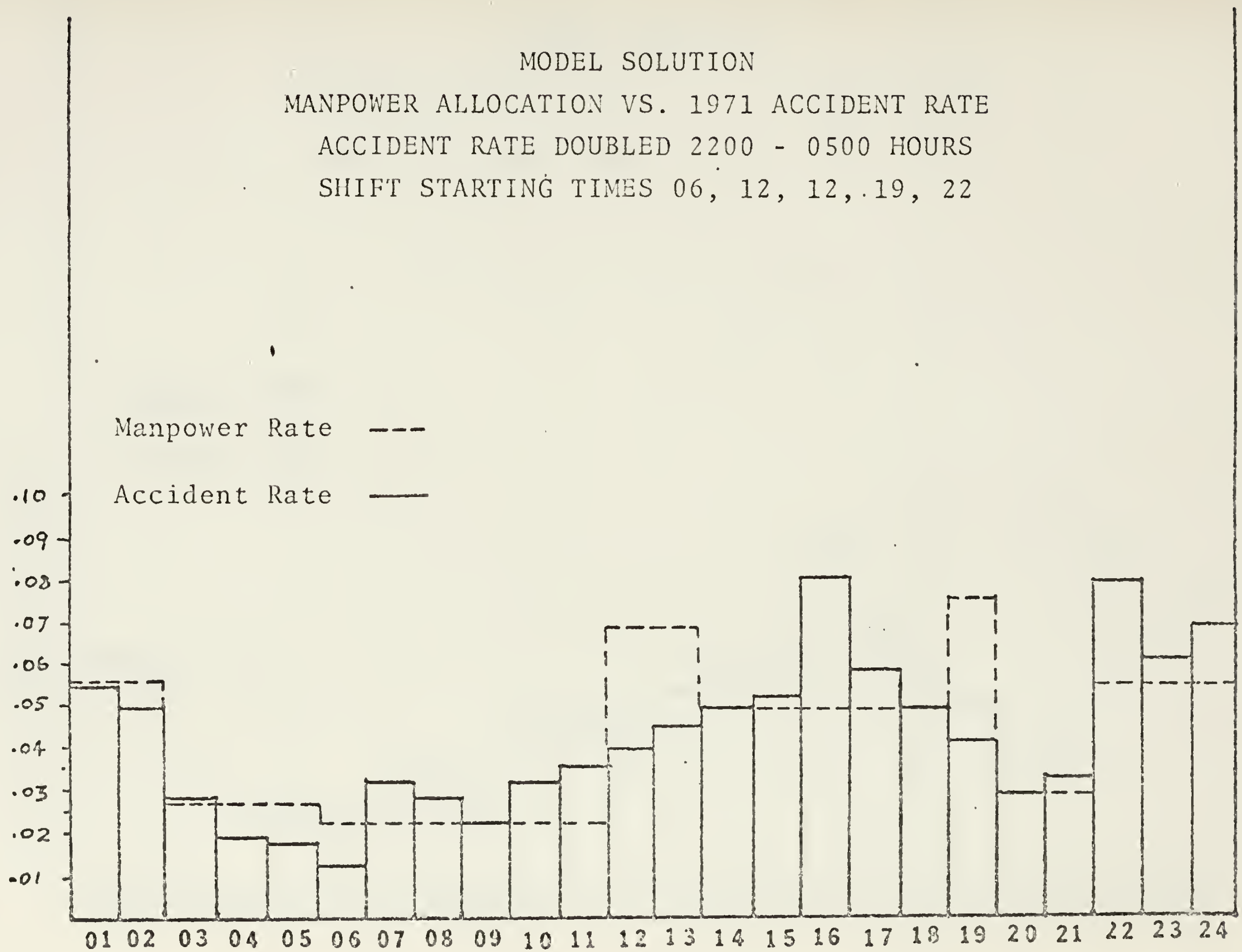

MODEL SOLUTION
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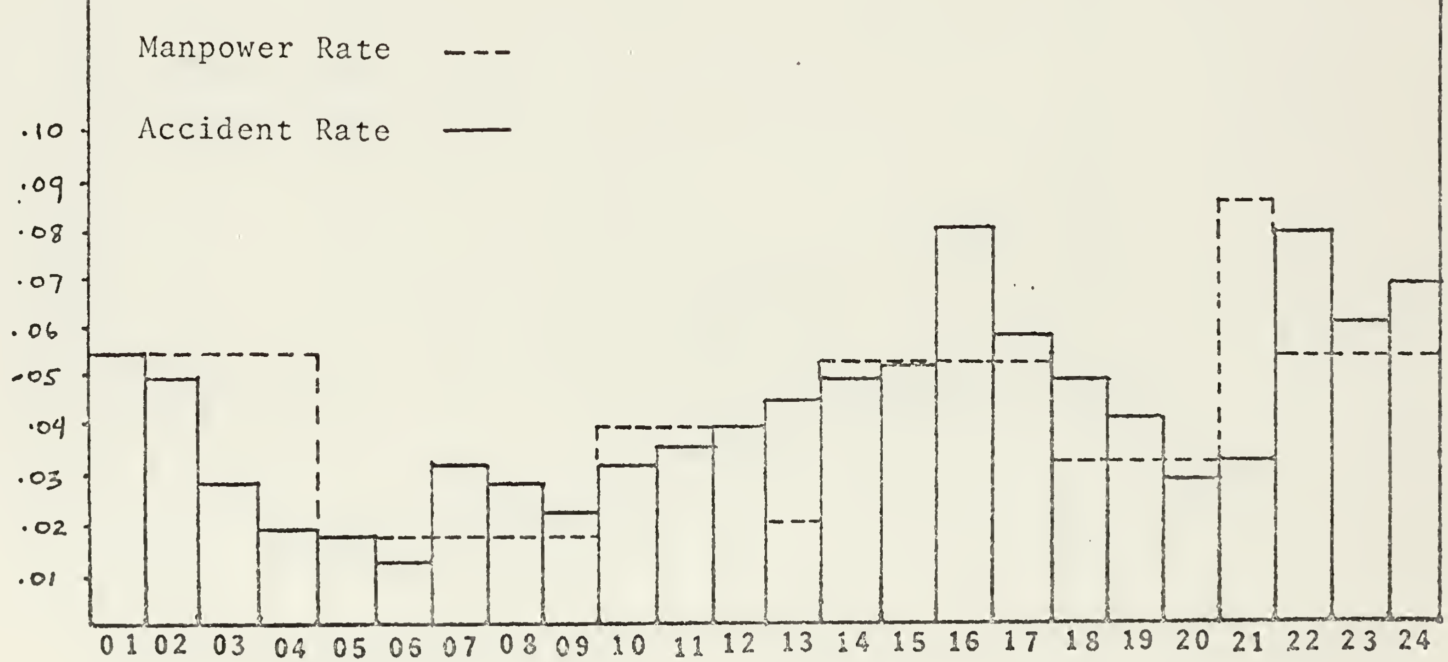
MODEL SOLUTION

MANPOWER ALLOCATION VS. 1971 ACCIDENT RATE

ACCIDENT RATE DOUBLED 2200 - 0500 HOURS

SHIFT STARTING TIMES 05, 10, 14, 21, 21

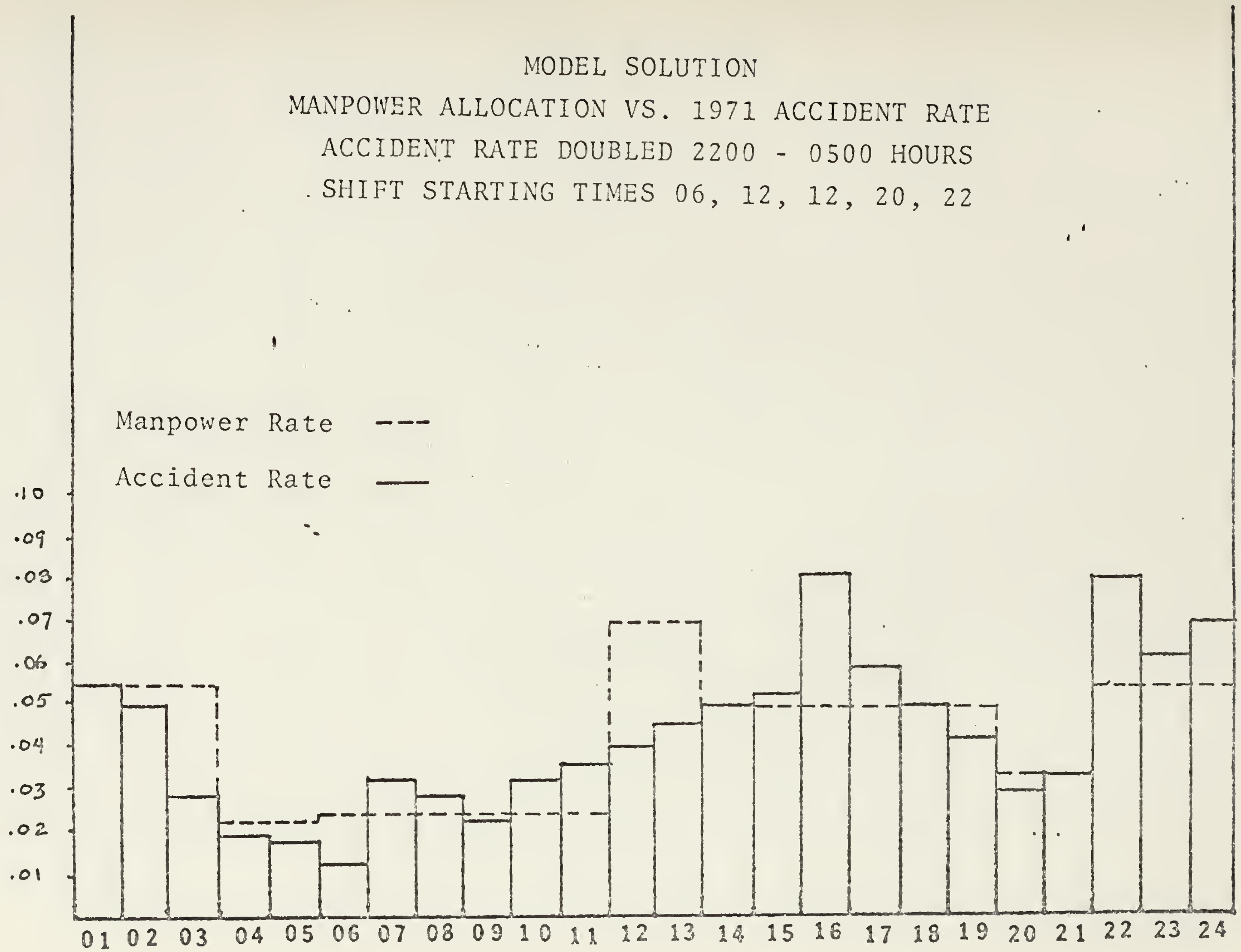


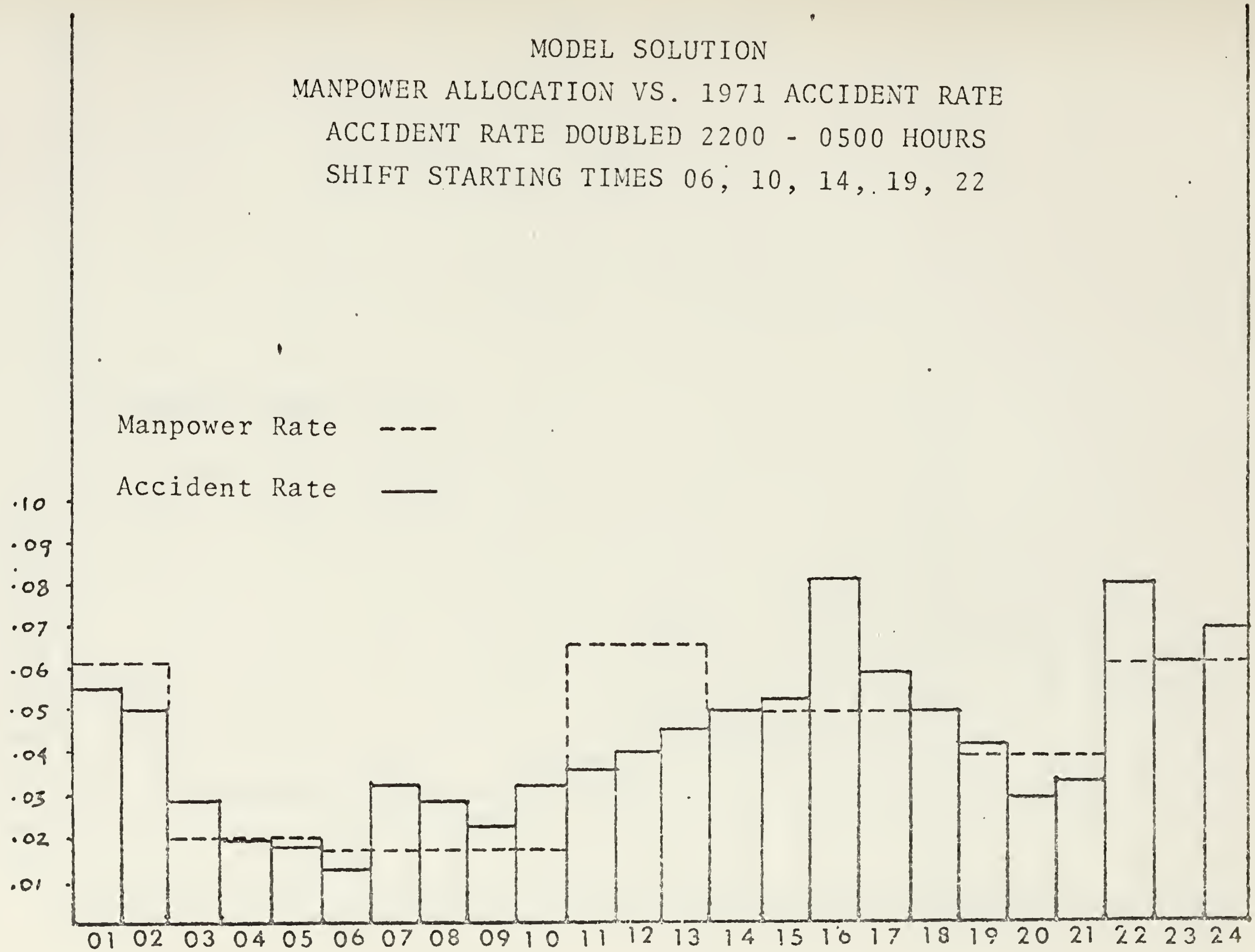
MODEL SOLUTION

MANPOWER ALLOCATION VS. 1971 ACCIDENT RATE

ACCIDENT RATE DOUBLED 2200 - 0500 HOURS

SHIFT STARTING TIMES 06, 12, 12, 20, 22




MODEL SOLUTION
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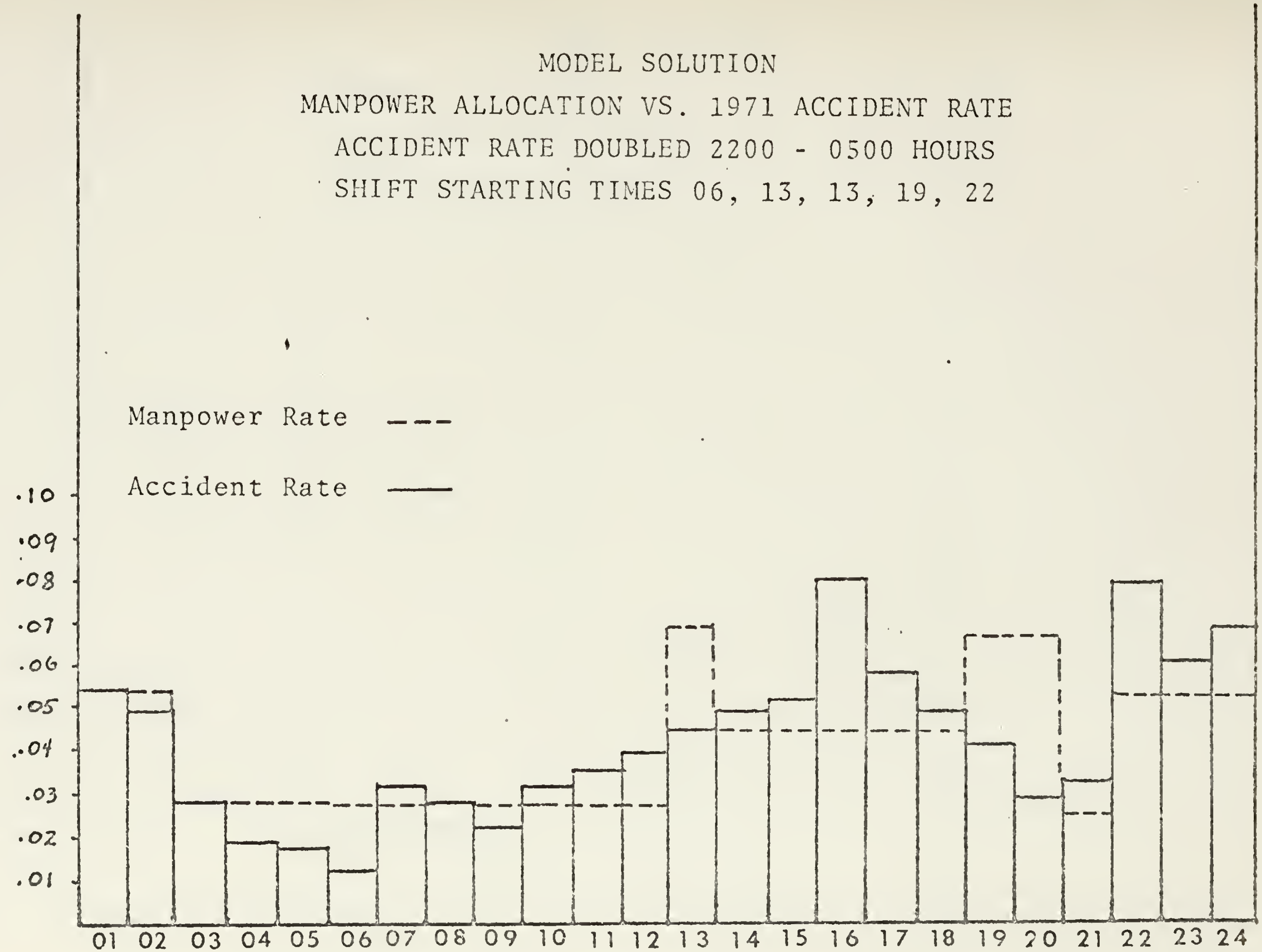
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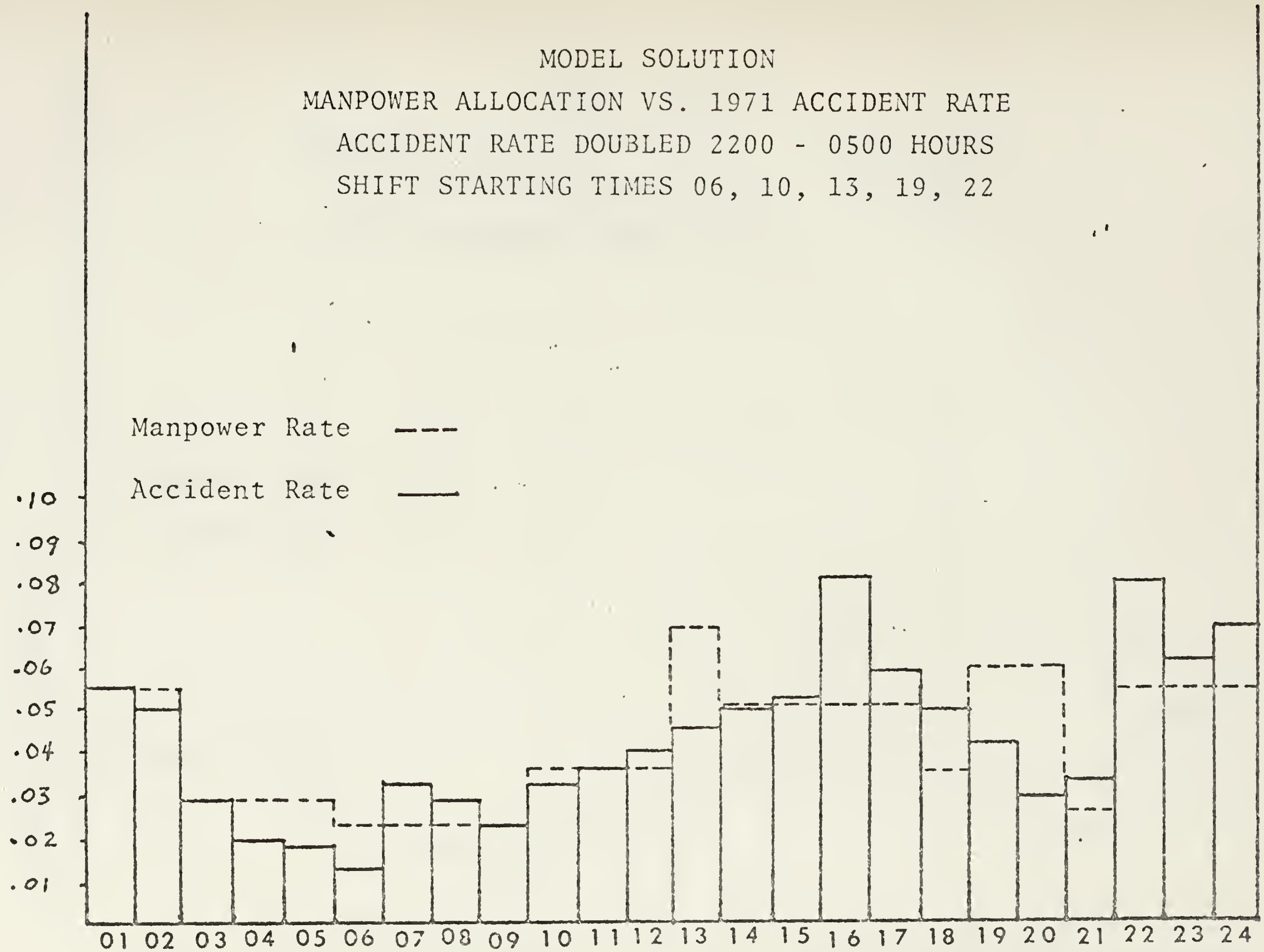

MODEL SOLUTION
MANPOWER ALLOCATION VS. 1971 ACCIDENT RATE
ACCIDENT RATE DOUBLED 2200 - 0500 HOURS
SHIFT STARTING TIMES 06, 10, 13, 19, 22
Manpower Rate
Accident Rate
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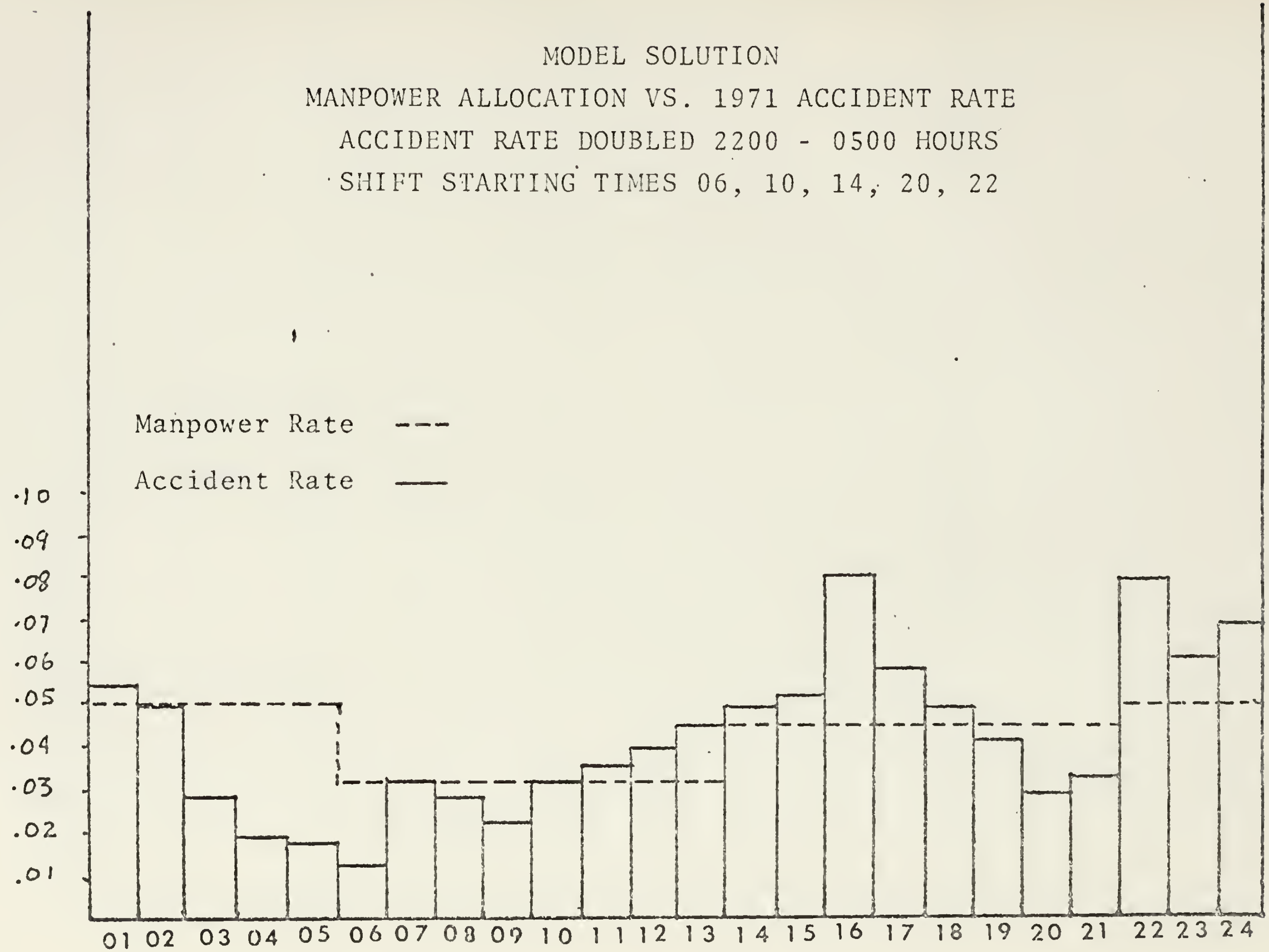
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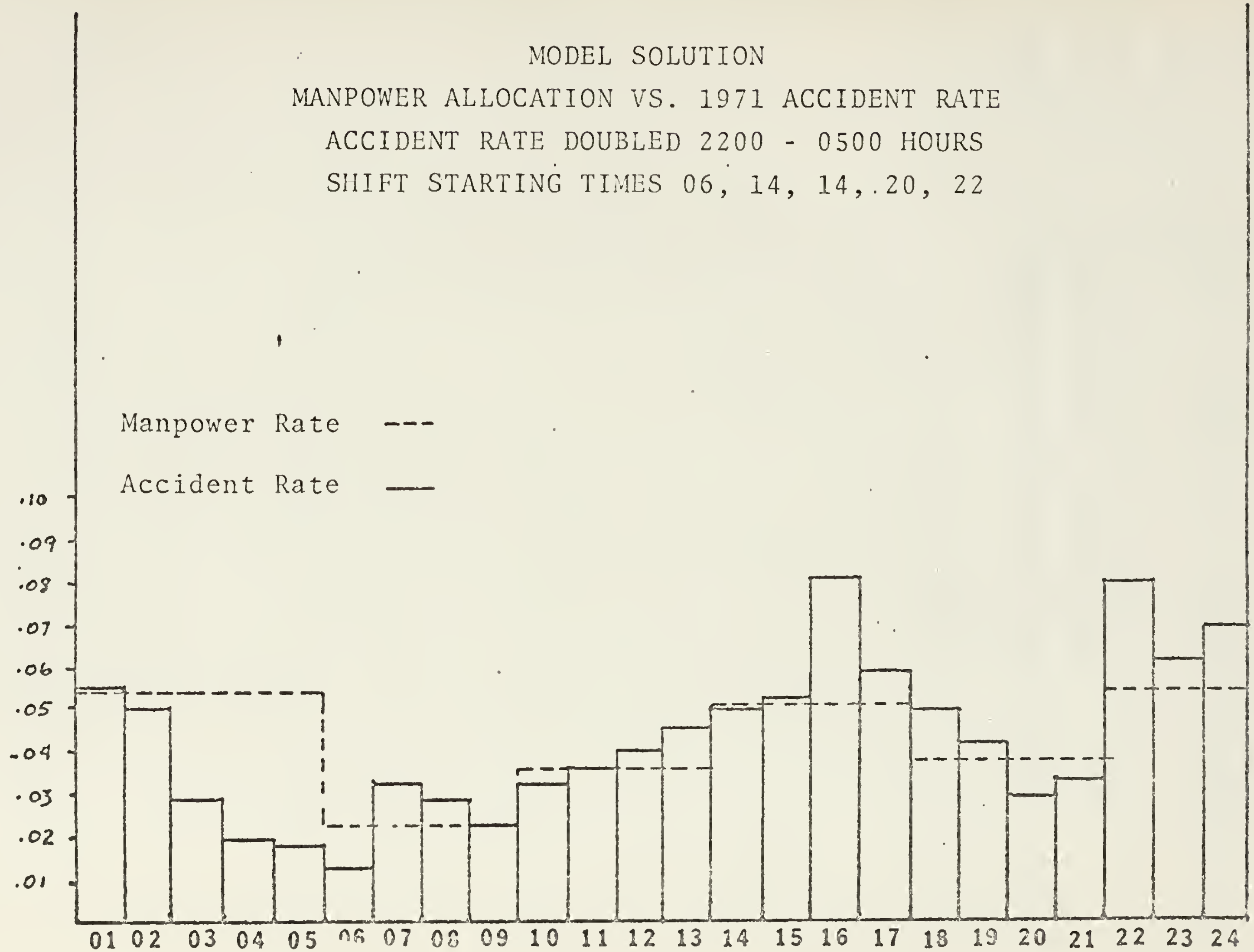
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01 02 03 04 05 06 07 08 09 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24

LIST OF REFERENCES

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ABSTRACT

This paper presents a mathematical approach to formulating problem in the allocation of manpower for the Monterey County Area of the California Highway Patrol (CHP). The technique employed is to formulate the manpower allocation problem of the CHP as a non-linear programming problem. The problem turns out to be separable, and the Mathematical Programming System/360 for the IBM 360 computer is subsequently used. This project was undertaken with the corporation of the CHP.

KEY WORDS	LINK A		LINK B		LINK C	
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Separable Programming Model						
Manpower Allocation						
Manpower Assignment						
California Highway Patrol						
Accident Rate						

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